Form is Matter

Triply periodic minimal surfaces structures by digital design tools

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Architecture and biology teach that the shape affects mechanical behaviour of structures therefore geometry is the basic concept of design, with an ethic responsible and sustainable approach, following the nature's organic model. Industrial design may apply formal properties of elementary shapes and basic design rules to manage the ``geometrical behaviour'' of new structural surfaces. The research aims to apply digital tools to the design of surface structures that maximise the matter efficiency in the development of ``solid fabrics'' with parametric controlled geometry.

Keywords: *Minimal surfaces, Parametric and generative design, Shape and form studies, Digital fabrication*

INTRODUCTION AND RESEARCH GOALS

Architecture and biology teach that the shape affects mechanical behaviour of structures, therefore geometry is the basic concept of design, with an ethic responsible and sustainable approach, following the nature's organic model. Industrial design may apply formal properties of elementary shapes and basic design rules to manage the "geometrical behaviour" of new structural surfaces. In facts nature's objects have been fundamental model since antiquity and the concept of Nature as a design model drives the theory of Architecture and the man's reference to natural forms a recurring statement in literature and it incorporates many basic design concepts, but the affirmation of digital technologies is changing the concept of *organic design*. Actually nature's patterns solve several project requirements, as it fulfils Alberti's *concinnitas* and the main architecture requirement, meaning Vitruvio's triad of *firmitas, utilitas, venustas*. [1]

Arts applied the reference in different ways: the first was the bare imitation in the pattern of ornaments and decoration, due to the admiration of the beauty of harmony and perfection in regular conformation in natural phenomena. Then Architecture applied skeletons' and trees' model to structures, imitating nature's balance in the proportioned relationship of building elements. Later it was the connection of parts in machines. Finally the imitation is fulfilled in the process of growth, which is the expression of life. In its digital procedures, the responsive Design copies vital process of life. Rules of basic shapes evolved in growth and form-finding processes.

The late development of digital technologies allows an important leap in the organic reference of design, improving the evidence of organic forms in design. *Imitation* is still the first way of learning, but *to imitate* does not mean *to copy*. In design it means *to reinvent*, therefore to understand and transform. Thus the imitation requires the very knowledge of form finding processes, which are due to a careful observation.

In the classical world the formal beauty was linked to a recognizable law that order the multiplicity in the unity: *symmetry, proportion* and *direction* resume rules that generate the shape starting from a *module*. Together they express the *eurythmy*, which in 1860 Gottfried Semper referred to in treatise *Der Stil*, as a '*concatenated sequence of spatial ranges, similarly shaped*'.

Everybody know what module means in architecture, and its importance about measurement, that is just ratio between quantity and unit; so this concept is directly connected with modular grids to control composition and proportion: thus design means measurement, which is geometry. The basic rules of form apply the same simplest operations of arithmetic and geometry: *addition, multiplication* and *division*. Growing and living processes implies *transformation*, that is changing in dimensions without changing topological ralationship in between elements, and/or responsive adaptation to external inputs. In computational design the concept of *module* plays the living principles of cell in organic fabric.

The biologist D'Arcy Thompson gave a wide explication of the geometry's evidence inside natural phenomena and architectures. Nature finds a static force balance in the symmetry of structures but in living beings it plays with different rules due to asymmetrical forces of growth, which imply dynamic transformation. He just stressed that life is tied to asymmetry and continuous transformation. [2] His work was fundamental to several architects, who pursued organic concept in architecture, such as B. Fuller and F. Otto. The first one applied the study of surface balance in cells to geodesic domes and the second designed light structures from the minimal surfaces' study with soap sheet and bubbles.

Minimal surfaces offer a great attraction to many disciplines. Some reasons for the common interest lie in the deep problems, which open up during closer investigation of their properties, and others in the widespread possible applications of minimal surfaces in completely different areas of research. Configurations of minimal surfaces have been found in a wide variety of different systems: from the arrangement of calcite crystals that form the exoskeleton of certain organisms to the theories that explain the nature of astronomical phenomena.

In design-building research, structures derived from minimal surfaces have led to the design of various typologies, such as tension-active roof structures, compression-active shells and large-scale architectural systems. This is, however, only one in between all possible uses. But in architectural structures, minimal surface structures remain rather unexplored for their suitable applications in design. Frei Otto in undoubtedly the main reference in the experimental development of minimal surfaces in light architecture, imitating self-formation processes in nature. He tried successfully application of the same efficient concept to different typologies of light architectures: tent structures, net constructions, pneumatic constructions, suspended constructions, shells, branched constructions and umbrellas as example of convertible constructions (moving and transformable). Actually Otto didn't *copy* the nature, but he referred to explaining it through technical developments. He stated that "*Technical object for which self formation process occur to a high degree form the natural border between natural and artificial*". [3]

Such as Frei Otto applied minimal surface theory in quite simple aggregation to architecture, as well mathematicians developed a larger set of surfaces from different boundary constraint with their further aggregation in modular lattices. *Triply Periodic Minimal Surfaces* (TPMS) are probably the ones that have the most interesting characteristics, including for design purposes. They are called periodic because they consist of a base unit that can be replicated, theoretically ad infinitum, in Cartesian space in three dimensions (triply), thus creating a new surface seamlessly and without intersections.[10]

Triply Periodic Minimal Surfaces, as it is visible in many natural systems, have a great potential, due to their structural efficiency thanks to overall area minimization, and efficient material distribution. Actually they comply Otto's requirements for *natural architecture*, because they apply/follow the nature's teaching in the balance of forces. [5] This is ethical and ecological approach, because it minimize the energy waste, saving material. [6]

Probably, despite at a theoretical level, the properties of Triply Periodic Minimal Surfaces have been investigated, the complexity of morphology has so far limited their use to manufacturability and design purposes.

The research aims to apply digital tools to the design of surface structures that maximise the matter efficiency in the development of "solid/permeable fabrics" through parametric controlled geometry.

This paper focuses on both the form-finding and the fabrication related to the geometric properties of TPMS. The aim is understand how the translation from the virtual three-dimensional space to the built artefact could be embodied into a computational process, which would also solve the issues within the fabrication framework

METHODOLOGY

The research follows two different paths, concerning basic topics:

- first, the definition of *computation tools*, meaning the selection of design parameters and the scripting of the digital form-finding process,
- next, their *design experimentation*, testing mechanical properties of different TPMS fabric.

Main computational design tools are:

- drafting formal and mechanical features according to shape;
- drawing basic shapes, by developing formal geometry features, starting from different minimal surfaces in a 3D modular lattice and developing modular aggregation by symmetrical repetition;
- selecting basic shapes that optimize the use of materials (minimal surfaces);
- defining tiles' parameters and lattices' aggregation rules, then to script formal codes and their transformation range;
- modelling selected modular lattices from basic geometry.

The design experimentation regards:

- to check formal behaviour of new structures in plane and in curved surfaces (flexibility, permeability, stiff movement and elastic responsiveness) and their adaptability to morphological transformation;
- to verify mechanical properties on printed prototypes (strength, lightness);
- to stress shape effects according to parameters variations, then optimize application range to different cases study.

The further development will inquire topics related to practical applications to case studies in design of printable objects, evaluating the adaptability to the object's shape, to their function and use as well to production requirements.

DESCRIPTION AND GENESIS OF MINIMAL SURFACES

A minimal surface is a surface whose mean curvature is always zero. This definition answers to the Plateau problem[11] proposed by Lagrange in 1760: if a closed polygon or oblique plane is assigned, then there is always a system of surfaces, including all possible surfaces that touch the frame, which are able to minimise the area. The minimal area of the soap film's surface of is one of the many examples that illustrates a well-known physical principle governing

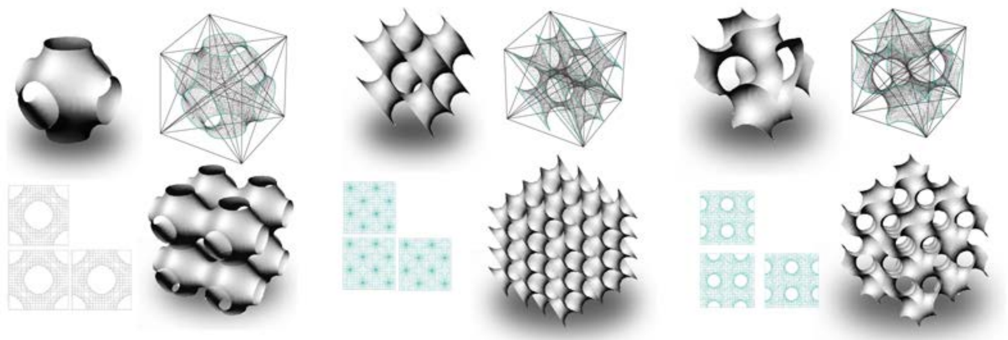


Figure 1
Triply Periodic
Minimal Surfaces

forms and motions of natural objects: the principle of least energy waste (or least action). It states that any physical configuration assumes its state or path in such a way that the energy requirement is minimal. In soap films, the shape minimizes the potential energy balancing the intermolecular force. Therefore this energy is directly proportional to the surface area of the soap films (assuming that the thickness of soap films is uniform) and, as a result, the soap films achieve minimal area.

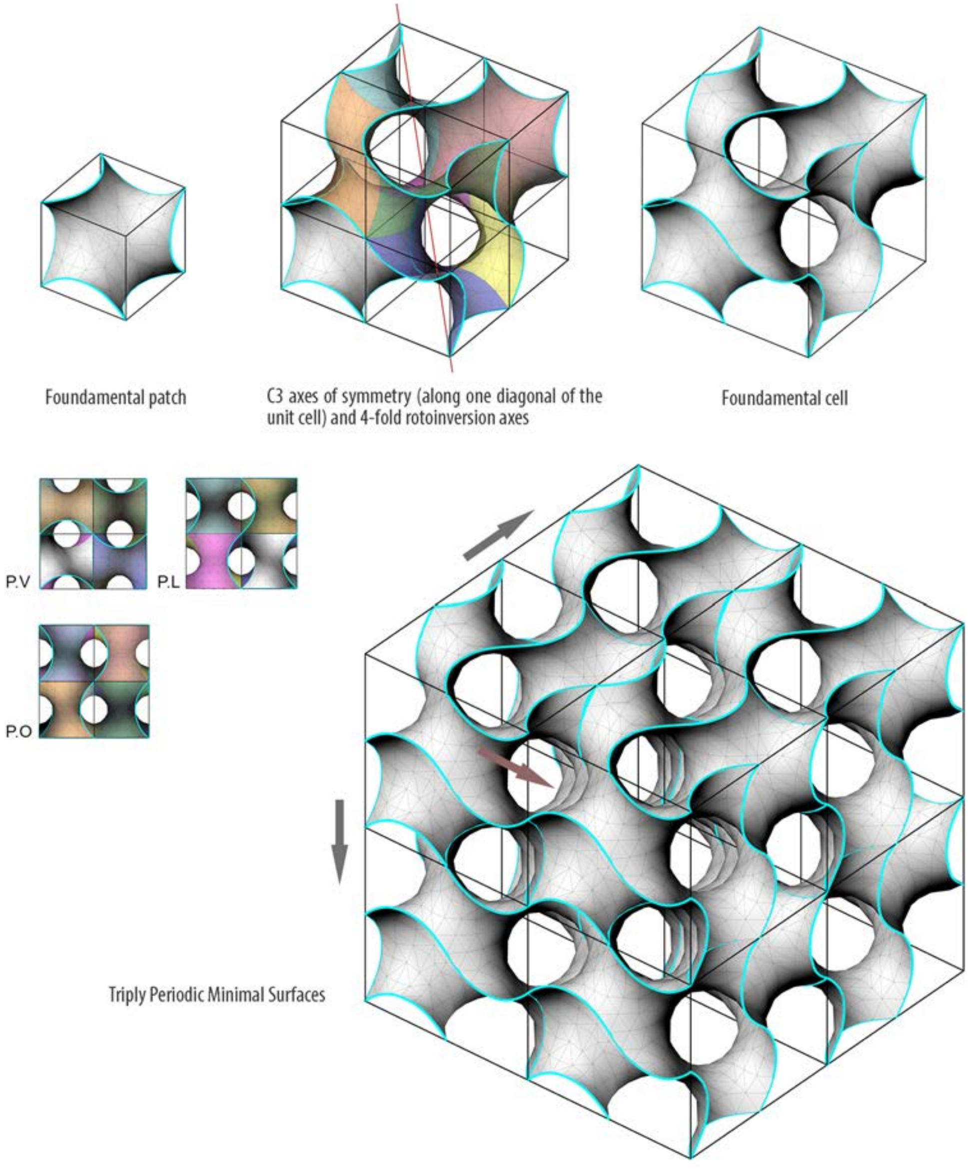
This means that minimal surface combine structure and material in a very efficient manner by aligning force and geometric form in an organic shape.

A triply periodic minimal surface (TPMS) is a minimal surface, which is periodic in three independent directions (Figure 1). TPMS are described in terms of a fundamental patch or asymmetric unit from which the entire surface may be built up by its symmetry elements. A single minimal surface is characterised by different curvatures: in other words, some surfaces are flatter than others. It follows that not all points of the surface support any concentrated loads equally well. If the same surface is, however, associated with a periodic distribution the physical iteration between the modules causes a compensatory effect that greatly increases their structural efficiency.

Because of that, the study of TPMS for design purposes is particularly fascinating (Figure 2). These surfaces may be made by defining and evolving their fundamental region, which is usually very simple due to the high symmetry, and then displaying many suitably transformed copies. Several fundamental regions are one of Coxeter's kaleidoscopic cells. Many of these surfaces were described by Alan Schoen in a famous NASA report. [10] The first step was to find a way to generate and control the TPMS in digital environment. The computation played an essential role in the simulation and modelling process of such complex phenomena. It was used Grasshopper, a graphical algorithm editor tightly integrated with Rhino's 3-D modelling tools in order to create an algorithm able to describe and to control various types of TPMS.

This research applies minimal surfaces that can be described by implicit form, typically a linear function of three variable, $f(x, y, z) = 0$. The trigonometric form is appropriate to the digital description because it allows the handling of the large number of elements that characterize TPMS, without overload the calculation process and also does not allow self-intersections. Using Grasshopper it's possible to define algorithms that are able to describe with good approximation any minimal surfaces directly from its implicit formulation. The algorithm translates the algebraic equation into a finished form that can be studied, manipulated and replicated.

Figure 2
Construction principles of TPMS based on gyroid



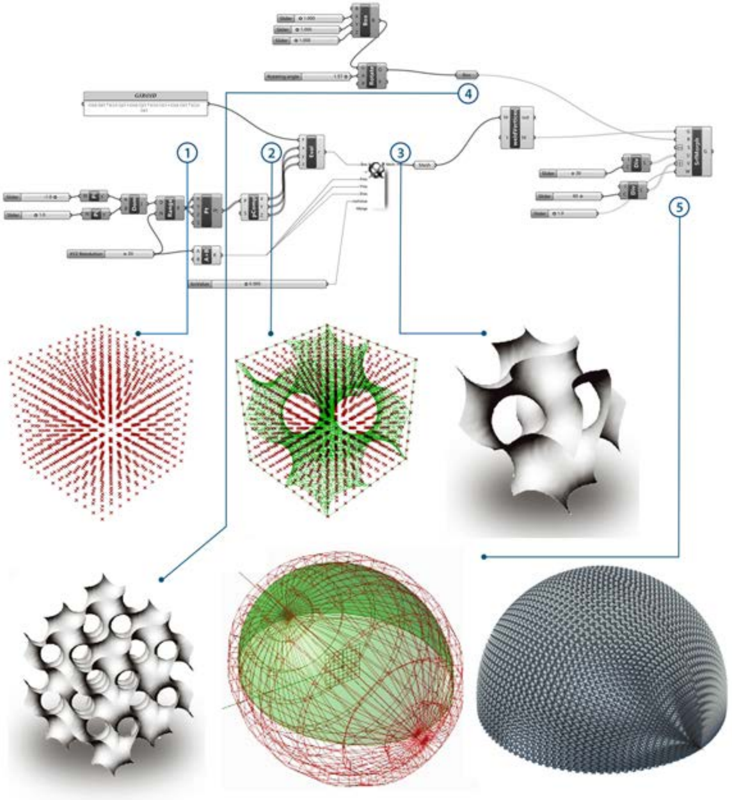


Figure 3
Step of the algorithm: 1) Definition of points in the fundamental cell; 2) Triangulation creates the surface; 3) Gyroid surface ; 4) Invariant translation to create a TPMS based on Gyroid; 5) Discretization of the hemispherical dome to obtain a surface composed by Gyroid.

The process can be conceptually simplified imagining that, in the domain of Cartesian space, the equation "selects" points, belonging to the surface you decide to represent (Figure 3). The next algorithm's instruction connects them by triangulation creating the surface. It is now possible to exploit the symmetry characteristics of the single unit by replicating it in a symmetrical cell, which is suitable to further replication in a modular lattice and to study the processes of adaptation to any required morphology. [7]

TPMS ANALYSES

The testing strategy applies the algorithm to a standard triply periodic minimal surface and it identifies the efficiency of the algorithm testing the level of accuracy in generating the geometry, comparing the two porous structures generated by *Gyroid* and *P-Surface*, in comparison with a solid bar.

To investigate the mechanical behaviour of different minimal surfaces structure, numerical simulations were also conducted. The model was implemented into finite element software code (COSMOS), which allowed the simulation and predicted the deformation characteristics of the designed porous structures and its mechanical behaviour, depending on the thickness changes.

A stress test was carried out (Figure 4). The application of 1kgf (10 N) was then evaluated on an iron parallelepiped sized 10x10x100 mm and compared the result with two equivalent-sized structures with different thicknesses, composed respectively of the P-surface and the Gyroid.

Table 1 summarizes main results.

This analysis leads to some data, which stress two main interest focuses.

First, in minimal surfaces element, even for very thin thicknesses (0.1 mm), the bar does not break even though it deforms considerably. This is because the stress does not focus on one point, but it is distributed among the different units that work in synergy: overall performance improves that of individual parts.

Second, the behavior changes with increasing thickness. When thick is 2 mm thick the deformation decreases considerably: if the solid iron bar deforms 1 mm, the one articulated in P-surface deforms 2 mm, while the Gyroid bar is deformed by 1.6 mm. The interesting aspect is that the two bars weigh, respectively, ten (P-surface) and eight times (Gyroid) less than the solid one, while maintaining good stress resistance properties.

Table 1
Stress test results

		THICKNESS (mm)	DISPLACEMENT (mm)	STRESS (Kgf)	WEIGHT (g)
	SOLID		0,00005	0,031	780
P-SURFACE	a	0,1	0,013	2,36	4
		0,5	0,0012	0,273	19
		1	0,0003	0,111	37
	b	2	0,0001	0,044	74
GYROID	c	0,1	0,016	1,1	5
		0,5	0,0008	0,40	25
		1	0,0002	0,115	49
	d	2	0,00008	0,055	98

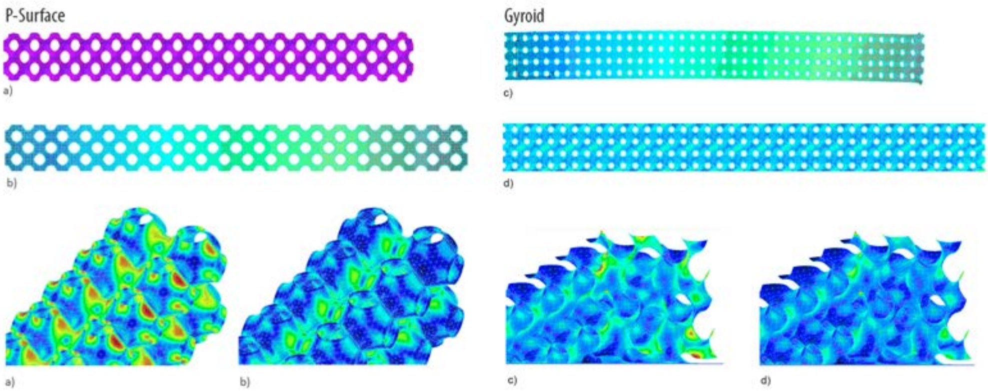
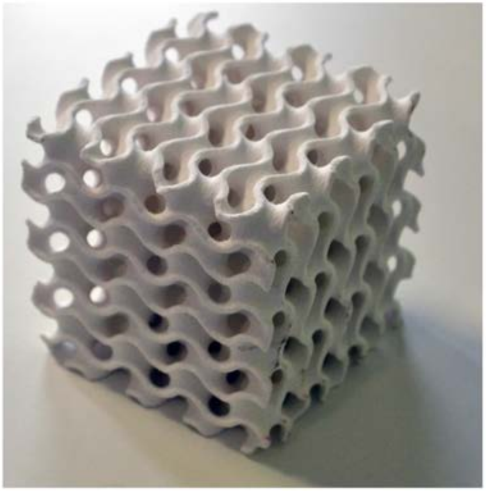


Figure 4
Stress test results



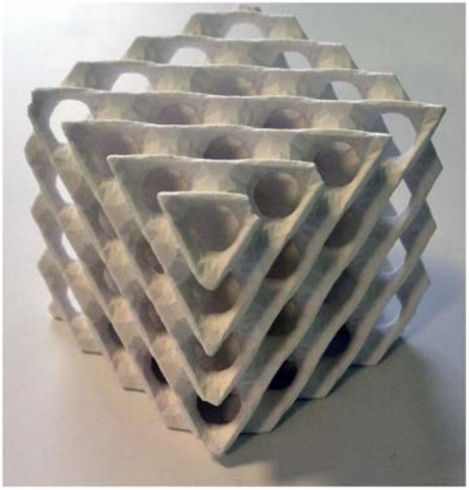


Figure 5
TPMS based on Gyroid and Diamond Surface manufactured by 3d printing process

FABRICATION AND TEST OF TPMS

The possibility of designing TPMS would be pointless if they couldn't then be created.

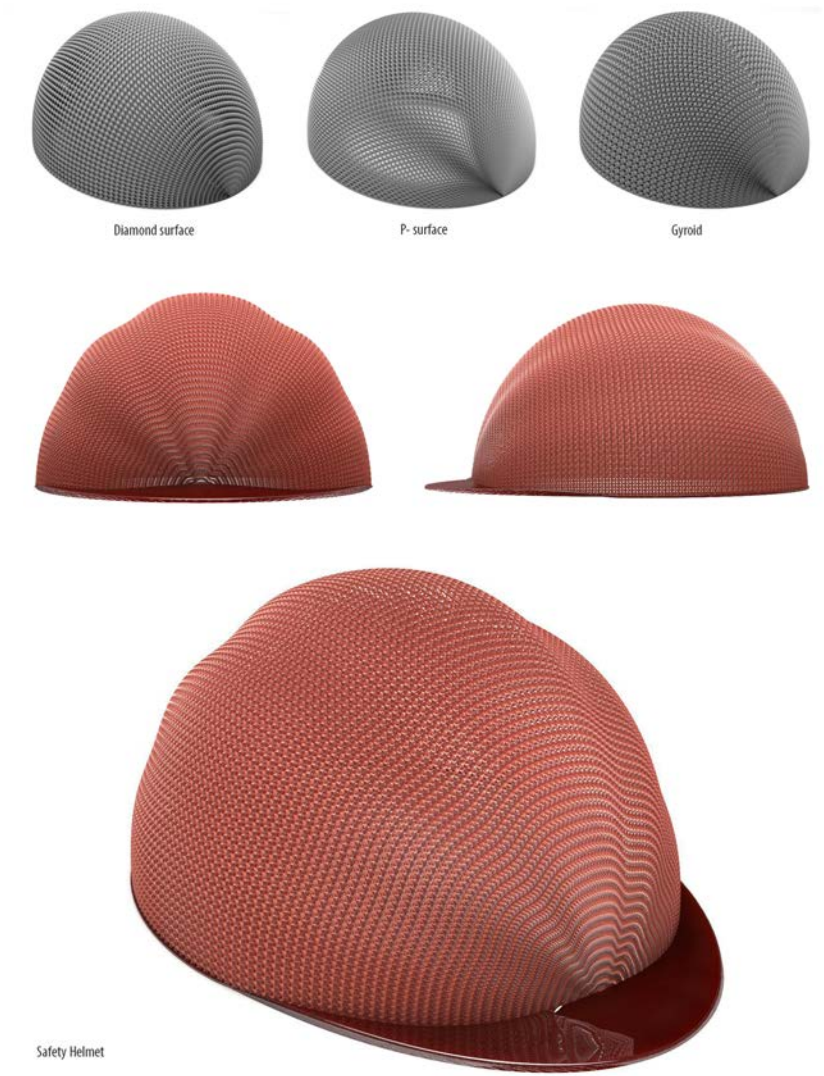
In recent years there has been a convergence towards the digitalisation of production processes thanks to machines able to construct, either in whole or in part, the designed object, starting from its digital model. This process is known as *Digital Fabrication* and does not require any additional interpretations to that of the designer, as the file is planned and the object can be fabricated without the involvement of other intermediaries. With other words, we could say that the scripting is both the *design representation* and the *making input*. Furthermore this new manufacturing permits the creation of forms and structures that were once considered extremely complex (Figure 5).

After assessing the cost-benefit ratio and research intentions, the first experiments were conducted with the *Z-corp Spectrum Z510* plaster-based 3D Printing. Considering that printers deposit layered material, moving vertically, usually it is necessary to provide the most correct arrangement to support the protruding parts to prevent the structure's collapse during the printing. It is worth noting that structures created using TPMS do not require additional support. If we consider that even in nature they are in processes where the generation takes place by layering, they are likely to be self-supporting body. This property makes it particularly suitable for 3d printing technologies, allowing a considerable saving of the production times and the construction material.

TPMS APPLICATIONS

A design application comes from an ergonomics research carried out simultaneously to this work: a study about safety of construction workers has shown as, especially in the summer, many operators do not wear helmet due to the weight and heat [7]. Furthermore, the three marketed sizes are not sufficient to meet the anthropometric variable, increasing the discomfort of many users. The example in figure 6 shows how, via computational modelling, it is possible to integrate the properties of the minimal surfaces into a protective artifact capable of solving these problems. After comparing the different minimal surfaces we chose to use the Gyroid-based Tpms. In nature, these structures are present where you need strength and lightness, such as in the sea

Figure 6
Comparison between the different Tpms and the ultimate safety helmet



urchin exoskeleton or butterfly wings. Stress tests performed (See TPMS analysis) confirm that the Gyroid is a structure that optimizes the ratio of used material, lightness and mechanical strength. These features, associated with the breathable characteristics due to the porosity of TPMS, make Gyroid particularly interesting for the design purposes proposed.

It has previously been explained how the algorithm allows to digitally describe the Tpms. It is now necessary to continue with scripting to adapt the fundamental cells to the protective helmet morphology

The helmet model was solved with a NURBS surface, discretized in parallelograms coinciding with the lower base of the pyramid trunk in which each single Gyroid will be recalculated. The limit of this procedure is the anisotropy of the fundamental cells that the hemisphere geometry involves. The mesh elements are more elongated in correspondence of the poles of the hemispherical dome. For compensation effects already mentioned, and for the small size of individual cells, this is not a problem from a structural point of view.

Assuming that future production costs for 3d printers will decrease,[7] it would be possible to obtain a helmet customized on anthropometric characteristics of the user, and that it is also lightweight, strong and which allows the circulation of air.

CONCLUSION

The study focuses on Triply Periodic Minimal surfaces and their structural system as a suitable manufacturing method. However, the potential of the suggested generative tool is not limited to these solutions, as the geometry could reach higher levels of complexity by exploring the design possibilities of all known periodic minimal surfaces or even to explore new types of surfaces or hybrid typologies.

The purpose of the research is to open a new direction within the computational design methodology, as part of design process, involving a multiple purpose design strategy, which takes into consideration various constraints, as part of an articulated parametric system.

Porous surfaces generated by TPMS could be interesting to various applications at different scale, from architecture to reach the level of industrial design artefacts, furniture or installations. Due to the cellular logical structure of the system, in correlation to the fabrication method, a feasible field of applications could include even fashion and textiles design.

So far is the beginning, concerning lightness and strenght, because several interesting properties are still to be tested: permeability, optical effect on color, sound absorption...The research must go on.

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