Fig. 1: Matter Design, *La Voûte de LeFevre*, Banvard Gallery, 2012.

LA VOÛTE DE LEFEVRE: A VARIABLE-VOLUME COMPRESSION-ONLY VAULT

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Particle-spring systems are commonly used to develop compression-only form-finding systems. This paper proposes to use a particle-spring system in response to a desired form in order to generate a variable-volume, compression-only structure fabricated of volumetric material. By varying the depth and the volume of the system, loads can be re-directed through the depth of material in order to result in a desired form, as opposed to a structurally optimal form that assumes a uniform thickness approach. This paper proposes to generate, build, and test a compression-only vault composed of variable-volume units. This research will advance knowledge surrounding volumetric physics calculations as well as volumetric fabrication methodologies.

INTRODUCTION

Thin-shell compression-only structural systems are relatively new to the built environment. Compression-only structures, on the other hand, are ancient. Thin-shell structures assume a minimal and consistent cross-section. This assumption is driven by material efficiency. The results are forms developed exclusively by structural concerns (typically gravity), hence the term form-found. Architecture has to respond to structural concerns, but it also has to address a variety of other issues, e.g. acoustical, formal, programmatic, etc. It is not necessary for form to be driven strictly by structural requirements. For example, Gothic cathedrals contain the thrust-vector within the variable depth of the stone's cross-section. These cathedrals are not determined by idealised catenary form, but through a confluence of architectural desires with compression-only principles. With this approach as inspiration, this paper addresses the potential of compression-only systems to be resolved through a variable volume in order to obtain a desired form.

Much research has been done in analysing existing variable-depth structures to determine if a thrust vector falls inside the depth of material.[1] Other methods assume a fixed depth of material in order to generate a design. The method proposed in this paper assumes a desired geometry and allows for a variable-volume to redirect the thrust vector as a means to produce a viable design that concerns both structure and other formal concerns. If typically one assumes thin, this paper assumes form.

This method is dedicated to addressing architectural concerns with structural results. This paper does not advocate for the reversion to a past architecture. It promotes the insertion of lost knowledge into our current means and methods of making.

PARTICLE-SPRING SYSTEMS

Particle-spring systems are based on lumped masses, called particles, which are connected to linear elastic springs. The solver used for this research is part of a particle-spring system implemented by Simon Greenwold.[2] 'Each particle in the system has a position, a velocity, and a variable mass, as well as a summarised vector for all of the forces acting on it.'[3] This Runge—Kutta solver is not necessary to generate a catenary (even load distribution), but it is necessary when evaluating an irregular load case. The method applied in this research will always be an irregular load case because it is assumed the resulting geometry is not an idealised catenary form.

Particle-spring systems have been explored to create virtual form-finding methods such as Kilian's *CADenary* tool.[4]

COMPRESSION-ONLY STRUCTURES

A compression-only structure will stand as long as the thrust vector of the system falls within the middle third of its cross section. It is not always predictable that a structure will fail, though it is possible to know if it will stand. A paper by Jacques Heyman introduced the safe theorem for masonry structures.[5] This theorem states that a compression-only structure can stand so long as one network of compression forces can be found in equilibrium within the section of the structure. This solution is a possible lower-bound solution. When evaluating existing structures, it is not always possible to understand exactly where this force network is.[6] The method applied in this paper can calculate and ensure a thrust vector falls within the thickness of material.[7]

FORM RESPONDING

Form-finding analogue models by such researchers as Otto[8] and Gaudi, or even the virtual versions like Kilian's *CADenary*,[9] have proved it is difficult to control and predict the results of the final found-form. Moreover, if that form does not correspond with a force that is external to the form-finding model, it is difficult to resolve the two into a solution. This paper proposes form-responding as approach. Form-responding takes a desired form as input and produces a variable-volume solution to allow for interaction between these external forces and the solver-based model.

METHODOLOGY

The vault is computed with a solver-based model that elicits a compression-only structure from a structurally non-ideal geometry. The model requires a fixed geometry as input and opens apertures in order to vary the weight of each unit. This dynamic system reconfigures the weight of the units based on a volumetric calculation. If unit A contains twice the volume of unit B, then unit A weights twice as much. It requires that the material of the project be consistent, and solid (hollow does not work). The computed result produces a project that will stand 'forever' as there is zero tension in the system precisely because of the weight and volume of the project, and not in spite of it.

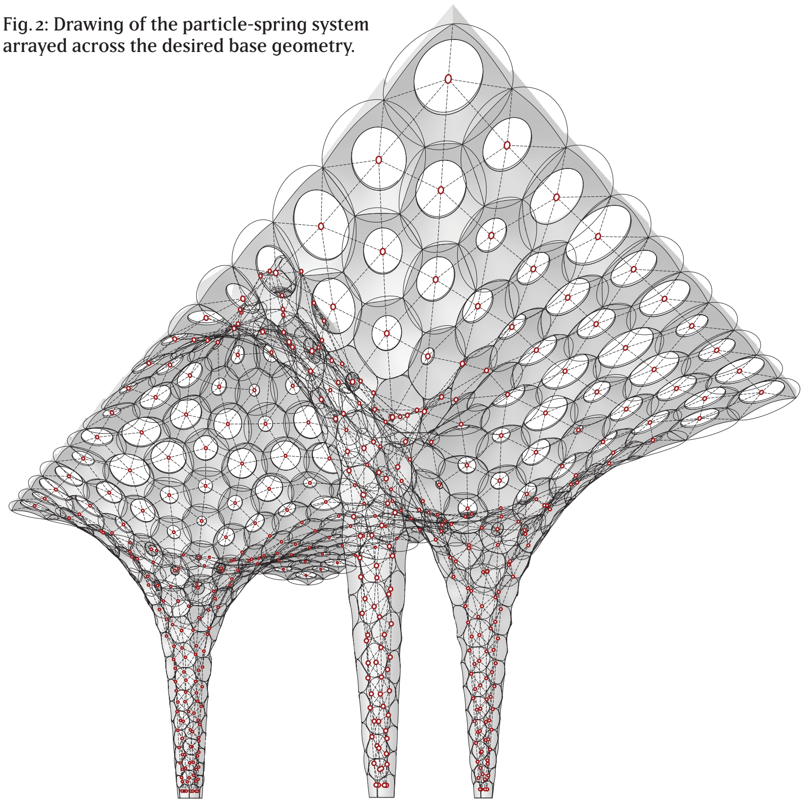
BASE GEOMETRY

This paper assumes the base geometry as fixed. The assumption is that this geometry has been predetermined by a force external to the model: acoustics, formal, building code, etc. Future research could allow for a more fluid and reciprocal relationship between the structural requirements and these other formal drivers. While this geometry is not strictly aligned with structural concerns, it must be close in order to result in a solution. In previous versions of the calculation,[10] almost any geometry would work as input. The variable-volume calculation is more nuanced.

This calculation requires a number of inputs to the system. It requires both an upper and lower bound surface. These surfaces parameterise the depth of the units as variable during the form generation, but fixed during the variable-volume calculation. The calculation also requires a location for the node of each unit to be located within the system.

These particles are evenly distributed across a base geometry that falls between the upper and lower bound surfaces. This distribution employs another particle-spring system to locate and distribute the points across the surface, increasing in distance from each other as they approach the upper elevations of the geometry. Figure 2 demonstrates the result: an enlarging of the units in the vault, and a tightening of the units down in the columns. The particle-spring system computes itself against these three inputs, which serve as the data.

Fig. 2: Drawing of the particle-spring system arrayed across the desired base geometry.



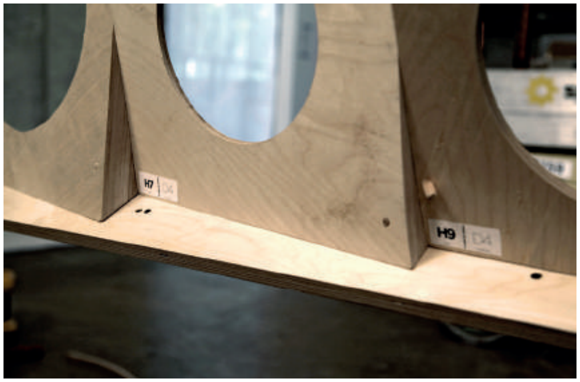


Fig. 3: Detail of the voussoir connection and indexing.

Fig. 4: The upper bound geometry skips continuity at the connection of the voussoirs due to the requirement for the milling operation to have a flat surface.

PARTICLE-SPRING SYSTEM

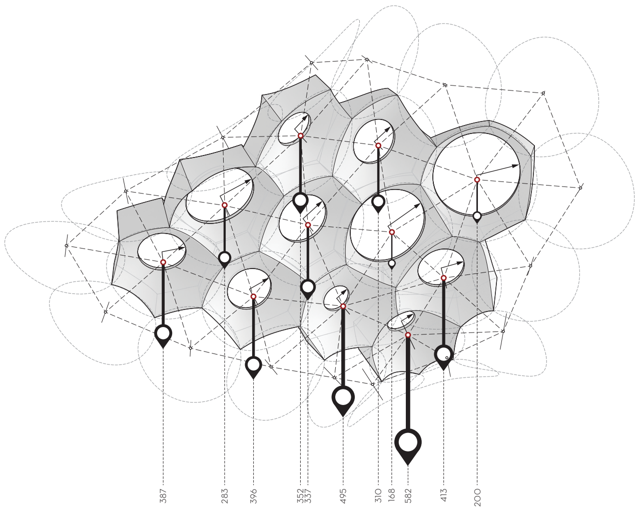
The particle-spring system is composed of a number of particles, the length of the springs that connect the particles, and the continual resulting forces on each particle informing the system. While the organisation is consistent, the system has been reconfigured in a variety of solutions.[11] This paper employs an evenly distributed system as described above.

VERTICAL DISTANCE VERSUS VOLUME

When analysing masonry arches, it is common practice to use static block analysis to break down an arch into a few polygons. The area of each polygon determines the vertical thrust vector.[12] Previous iterations of this calculation employed a high resolution of vertical distances to inform each particle with its new relative weight. This paper employs volume as opposed to area or distance. Similar work has been conducted using volume to analyse and determine the viability of a structure.[13] This paper employs the variability of the volume to ensure a solution.

The location of the particles defines the virtual thrust network. In order to ensure a solution, these particles are required to be moving during the calculation until they find equilibrium. At each interval of the calculation, a number of operations occur, complicating the calculation beyond a simple distance measurement. The new location of each particle generates a three-dimensional Voronoi calculation that intersects with the lower bound base geometry surface. This intersection then produces points at the intersection of each curve where an interpolated curve is generated. Simultaneously, the centroid point (also the particle) finds the closest points on the upper bound surface and generates a circle perpendicular to the line connecting these two points. The plane this circle is generated on also serves as the flat backside that sits on the table of the computer numerically controlled (CNC) router, a useful fabrication constraint (see figs. 3, 4) The circle and the curve are then lofted with each other, producing a surface that is trimmed with the rest of the surfaces in the system. The intersection of these surfaces extrudes to the closest position on the upper surface, producing the voussoir[14] that discretises each unit in the vault.[15] Each unit now contains an enclosed volume that can inform the system with its weight relative to its neighbours. Figure 5 demonstrates these operations. These operations are calculated continually until the system finds equilibrium and a solution can be detected.

Fig. 5: Diagram of particle-spring system and the variable volume calculation. The volume of the enclosed surfaces equals the vertical thrust on the particle.



DESIGN

A deliberate attempt was made in this project to topologically[16] transition from column to vault. No break is inserted in this transition; however, this is a lie. In reality, there is a difference between column and vault. The column is solid (fig. 6). It is treated as a single unit. The vault on the other hand is discretised into its constituent units.[17] This moment of discrepancy is attempted to be seamless; however, the grain of the wood demonstrates the reality. There is a good reason for this false reality. A column does not perform in the same manner as a vault. The thrust vectors inside the column are vertical, not progressively horizontal. To that end, a column does not resist horizontal thrust. It resists buckling. The solidity of the column is paramount.

The discrepancy in transitioning from solid column to discretised vault is resolved via rhetoric. The rhetoric of individual units continues down the column as if the single and solid column was in fantasy an impossible continuation of the units to the ground. This rhetoric is not a simple continuation of the conical-Boolean geometry that composes the vault. It is a new, yet similar approach. It refers to the conical-Boolean, without repeating it. This shift in geometry allows the system not only to calibrate volume (as applied in the vault), but also to perform another transition from fragmented to smooth. As the units make their way down the column, they do get smaller, but the dimples slowly make their way to the surface, producing the illusion of continuity, only to push through that continuity as the very base. This punctuation to the statement suggests that the weight of the vault above is so great that the column is forced to bulge outward.

Fig. 6: Column detail, Matter Design, *La Voûte de LeFevre*, Banvard Gallery, 2012.

FABRICATION

The vault was produced with Baltic birch plywood. The plywood is sourced in three-quarter-inch thick sheets awaiting the 'thickening'. Perhaps it is evidence of the state of the industry that volumetric material is difficult to procure. Each custom unit is digitally dissected and sliced into these thicknesses, cut from the sheets, and then physically reconstituted into a rough volumetric form of their final geometry. These roughs are indexed onto a full sheet and glued, vacuum-pressed, and replaced onto the CNC router as demonstrated in figure 7. This process is materially more efficient than carving these units from one solid block of material, though it is more laborious.

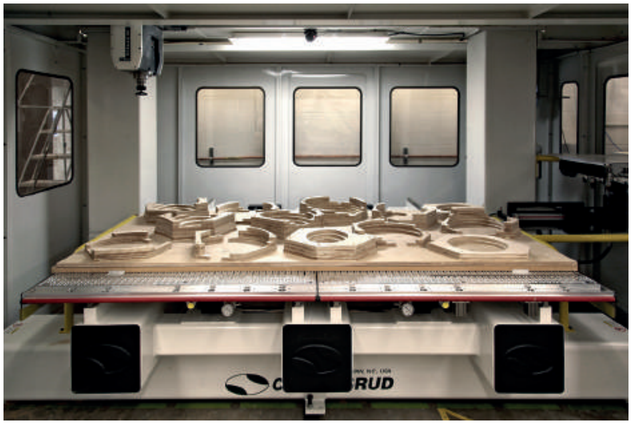


Fig. 7: Roughed aggregated blanks of the desired geometry await the milling operation on the five-axis machine.

Fig. 8: Swarf milling the voussoir edges.

This project is produced on a five-axis Onsrud router.[18] The swarf [19] toolpaths utilised are dedicated to removing the most material with the least effort (fig.8). Instead of requiring the end of the bit to do the work, this path uses the edge of the bit to remove much more material. Because this method traces the geometry with a line, as opposed to a point via Philibert De L'Orme's technique stereotomy,[20] it requires the units are constituted of ruled surfaces.[21] This constraint informed the conical-Boolean geometry in the vaulted portion of the project, though relaxed in the columns where a more typical surface milling operation produces the rhetorical bulges. This shift in tooling operation also speaks to the understanding of the difference between column and vault.

ANALYSIS

This project was fabricated with an assumed zero-fill approach. As part of the requirement that the vault must be dismantled, there is no mortar. Discrepancies, errors, and gaps were impossible to resolve because of this zero-tolerance approach. In order to ensure completion on site in difficult locations, a manual band saw handled the work of removing collision material on the backside of the problematic units. This on-site carving did not affect the front edge of the units, but it did produce a gap where the voussoir surfaces were not coincidental. This happy accident aligns precisely with the Inca wedge [22] process, where masons would fill from the backside of a wall with mortar into a voided wedge between stones, while the front and

Fig. 9: Array of all the unique voussoirs that compose the vault.

Fig. 10: Various unique voussoirs that compose the vault.

Fig. 11: Assembly of the vault.

Figs. 12, 13: Matter Design, *La Voûte de LeFevre*, Banvard Gallery, 2012.

architectural face appeared to be mortarless. There is room for further exploration to capitalise on the potential of the Inca wedge method.

CONCLUSION

La Voûte de LeFevre demonstrates the potential of informing contemporary fabrication methodologies with past knowledge concerning volume. It successfully employs physics simulation to ensure stability through volumetric calculations that serve in reciprocity with volumetric making processes. While aggregate Baltic birch plywood serves as an analogue, potential is seen in other volumetric materials, such as autoclave aerated concrete, plaster, or stone.

ACKNOWLEDGEMENTS

This paper presents results of an ongoing research project that began at the Princeton University School of Architecture under the tutelage of Axel Kilian and continued at Ohio State University, Knowlton School of Architecture (with Howard E. LeFevre Emerging Practitioner Fellowship funding) and the Massachusetts Institute of Technology (with Belluschi Lectureship funding). Two particle-spring systems have been used. The first was implemented by Simon Greenwold in Java as a library for 'Processing' (www.processing.org), an environment developed by Ben Fry and Casey Rease. gHowl (www.grasshopper3d.com/group/ghowl) by Luis Fraguada was used to communicate via UPD between processing and Grasshopper (www.grasshopper3d.com), a plug-in developed by David Rutten for Rhinoceros (www.rhino3d.com), a program developed by Robert McNeil. The second particle-spring system was generated entirely inside Grasshopper with the aid of two plug-ins: Kangaroo (www.grasshopper3d.com/group/kangaroo) by Daniel Piker served as the physics simulation of the particle-spring system and Hoopsnake (www.volatileprototypes.com/projects/hoopsnake) by Volatile Prototypes allowed the vertical distance to loop back into the calculation.

NOTES

1 Philippe Block, Thierry Ciblac and John Ochsendorf. 'Real-time limit analysis of vaulted masonry buildings', *Computers and Structures*, 84 (2006), pp. 1841–52.

2 The first particle-spring system was implemented by Simon Greenwold in Java as a library for 'Processing' (www.processing.org), an environment developed by Ben Fry and Casey Rease. Greenwold, Simon and Edward Allen. 'Active Statics', 2003, accessed 1 November 2011. http://acg.media.mit.edu/people/simong/statics/data/; Ben Fry and Casey Reas, 'Processing web site' (2011), accessed 1 November 2011. http://www.processing.org/.

3 Axel Kilian and John Ochsendorf, 'Particle-Spring Systems for Structural Form Finding', *Journal of the International Association for Shell and Spatial Structures*, 46/2 (2005), pp. 77–84.

4 Ibid.

5 Jacques Heyman, 'The Stone Skeleton', *International Journal of Solids and Structures*, 2 (1966), pp. 249–79.

6 Philippe Block and John Ochsendorf, 'Lower-Bound Analysis of Unreinforced Masonry Vaults' in Dina d'Ayala and Enrico Fodde, eds., *Proceedings of the VI International Conference on Structural Analysis of Historic Construction* (London: CRC Press, 2008), pp. 593–600.

7 For further reading on lower-bound analysis of unreinforced masonry structures: Jacques Heyman, *The Masonry Arch* (Chichester: Ellis Horwood, 1982); Santiago Huerta, 'Mechanics of Masonry Vaults: the Equilibrium Approach', in Paulo B. Lourenço and Pere Roca, eds., *Historical Constructions 2001, Universidade do Minho, Guimarães: Possibilities of Numerical and Experimental Techniques* (Coimbra, Portugal: Institute for Sustainability and Innovation in Structural Engineering, 2001), pp. 47–70. Santiago Huerta, *Arcos, Bóvedas y Cúpulas: Geometría y Equilibrio en el Cálculo Tradicional de Estructuras de Fábrica* (Madrid: Instituto Juan de Herrera, 2004).

8 Frei Otto and Bodo Rasch, *Finding Form: Towards an Architecture of the Minimal* (Berlin: Edition Axel Menges, 1995).

9 Kilian and Ochsendorf (see note 3).

10 Brandon Clifford, 'Thick Funicular: Particle-Spring Systems for Variable-Depth Form-Responding Compression-Only Structures', in Xavier Costa and Martha Thorne, eds. *Change Architecture Education Practices: 2012 ACSA International Conference* (New York: ACSA Press, 2012), pp. 475–81.

11 Brandon Clifford, 'Thicker Funicular: Particle-Spring Systems for Variable-Depth Form-Responding Compression-Only Structures', in Paulo Cruz, ed. *Structures and Architecture: Concepts, Applications and Challenges* (London: CRC Press, 2013), pp. 205–6.

12 Block and Ochsendorf (see note 6).

13 Block and Ochsendorf (see note 6).

14 Voussoir: a wedge-shaped element, typically a stone, used in building an arch or vault.

15 The surface geometries enclosing this volume are generated with ruled surfaces due to a reciprocal relationship with the method of fabrication. For more information on this process, see note 6.

16 Topology: in mathematics, the study of the properties of a geometric object that remains unchanged by deformations such as bending, stretching, or squeezing but not breaking.

17 A similar strategy of the solid column transitioning into voussoirs above was employed in Peterborough Cathedral. These voussouirs also misalign on the upper bound geometry, while aligning precisely on the lower bound (visible surface). For more information, see Robin Evans, 'Drawn Stone', in *The Projective Cast: Architecture and Its Three Geometries* (Cambridge, Mass.: MIT Press, 2000), pp. 178–239.

18 With fabrication support from the University of Michigan Taubman College FABLab.

19 Swarf machining is a technique that allows side cutting with an end mill while proceeding along the surface of a part, such as the sidewalls of a tapered rib.

20 Philibert de L'Orme (sixteenth century) was, like Palladio, the son of a mason. He merged into architecture, not through a series of rigorous understandings of form or technique, rather from the builder or mason. In his printed work of 1567, *Le Premier tome de l'architecture*, Philibert de L'Orme introduced the method and definition of *art du trait géométrique*. This method developed as a way to reciprocally draw what can be built and vice versa. Because of this emergence, de L'Orme can also be credited as the first professional architect, because his technique served to instruct and communicate between the designer and the builder, though an important distinction should be drawn between the representations of architecture we now generate and de L'Orme's descriptive geometry that served as method template to construction. In a way, de L'Orme can be considered the predecessor to digital fabrication. For more information on this topic, see Evans, 2000 (see note 17) and Brandon Clifford and Wes McGee, *Range: Matter Design* (Reykjavik: Oddi, 2013); as well as Matthias Rippmann and Philippe Block, 'Digital Stereotomy: Voussoir Geometry for Freeform Masonry-Like Vaults Informed by Structural and Fabrication Constraints', in *Proceedings of the IABSE-IASS Symposium* (London, 2011).

21 This project is part of a line of work dedicated to this proposal of employing the line for carving. For more information, see: Brandon Clifford and Wes McGee, 'Periscope Foam Tower' in Ruairi Glynn and Bob Sheil, eds. *Fabricate: Making Digital Architecture* (Cambridge, Ont.: Riverside Architectural Press, 2011), pp. 76–9.

22 Brandon Clifford, *Volume: Bringing Surface into Question* (SOM Prize Report, SOM Foundation, 2012), pp. 286–9.