

# **A Geometry Exploration of Flexagons: Designing a Tetrahedron Based Responsive Daylight Control System**

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**Abstract.** This project aimed to expand the area of responsive shading systems through the novel application of a volumetric origami geometry – the flexagon. The original contributions of this project come through the design development and prototyping of the kinetics of an octa-flexagon based geometry. Few researchers or designers have investigated the flexagon pattern in architecture and departing from relevant research, this project identified a novel geometric construct of flexagons that allow kinetic actuation with beneficial performative and aesthetic properties. These include surface qualities of the component tetrahedron geometry for daylighting and view control. The aggregation of multiple units resulted in new understanding of the stacking characteristics and the rotational envelope of flexagon geometries.

**Keywords:** Architectural Geometry, Prototyping, Origami, Responsive Façade.

## **1 Introduction**

Facades are a building's first line of defense against climate and weather conditions. There are many different approaches that buildings currently use in order to combat climatic extremes. Some buildings offer solar shading to help with solar gain, some take advantage of natural ventilation to keep the building cool. From a human comfort and energy efficiency standpoint, shading design considerations for facades include optimizing daylighting and views, controlling heat gain and loss. Generally, conventional static shading systems are designed to reduce solar energy gain in the summer and to take advantage of solar energy during the winter while achieving optimal daylighting conditions. While the solutions work very well on average, they cannot accomplish optimal building performance for changing seasons or daylighting [1]. Every season has its own requirements for a façade. Some variables for the hot months include, natural ventilation, solar shade, and some insulation value to

maintain cool temperatures. Requirements for the winter include tight seal with no ventilation, solar gain for heating, and a higher insulation values.

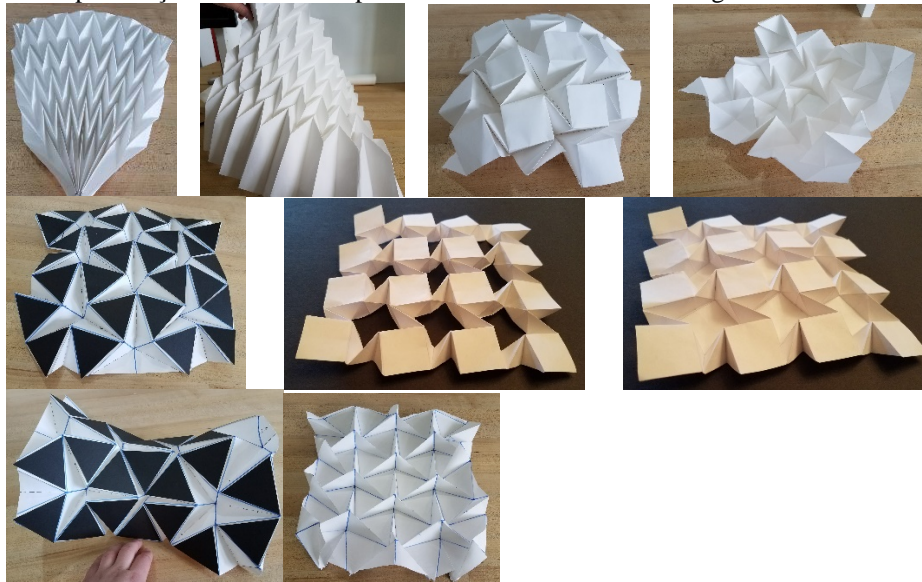
With technological advancements, a number of facades or prototypes with kinetic or responsive capabilities have been designed and developed. Such systems allow to adapt to changing environmental conditions and user needs over daily or seasonal cycles. For example, starting with Buckminster Fuller's Montreal Expo '67 Dome with automated louvers, Institute Monde de Arab aperture panels, the Abu Dhabi Investment Council Headquarters shading panels, the Royal Melbourne Institute of Technology (RMIT) Design Center façade, and Barcelona Media-TIC building inflatable ETFE cushions, highlight environmentally responsive façade systems [2]. At the experimental installation scale and smart material level, Do|Su Studio Architecture experimented with thermos-bimetal self-ventilating surfaces, or Decker Yeadon proposed a homeostatic façade system integrating engineered dielectric elastomers-based ribbons sandwiched into a double-skin glass façade that adapt to sunlight and temperature variations. Velikov and Thun [3] discuss the need for standardized terminology that defines evolving field of high-performance responsive building envelopes that can be characterized by the concepts of smart, intelligent, interactive, and responsive.

## **2 Flexagon Geometry**

Rigid origami geometry have inherent properties of structural stiffness, expansion and contraction, and surface reconfigurability. Filipov et al. [4] developed zipper-coupled tubes which permit only one flexible deployment of a tubular structure. The versatility of such systems are proposed for deployable structures in robotics, aerospace, and architecture. Several researchers have investigated folding with planar surfaces geometries ([5], [6]) for large span roof surfaces or parametric ventilation surfaces. Only a few examples of volumetric three-dimensional folding applications have been researched for architectural applications. Elgahzi et al. ([7], [8]) study through digital simulations the shading performance of so-called Kaleidocycle rings for shading purposes and suggest that those can enhance daylighting performance. However, their investigation remains in the digital realm and does not address the kinetic or architectural possibilities of this type of system. Burton [9] explored and designed a three-dimensional flexagon system composed of a series of tetrahedrons that emulate floors, walls, and ceilings to allow for spatial reconfiguration. While Burton's study involve both digital simulations and physical prototyping his thesis work focuses on spatial building scale interventions in comparison to shading systems. Based on the authors literature review at the time of preparing this paper, there is no research yet that has further investigated the kinetic and architectural implications of 3D flexagons for daylight control purposes. The authors explain the design development and prototyping process of a flexagon based shading system, and discuss the physical constraints and architectural opportunities this system offers.

### 3 Design Experimentation and Prototyping

This project started with the intent to explore surface deformations using computational tools such as Rhino/Grasshopper as well as physical model studies with a variety of different materials. Initial explorations began with origami and other flexible surface techniques to get a better understanding of the range of motions and the level of deformation the surface would achieve (Fig. 1). This helped to understand surface curvatures and how different angles and directions of the folds affect the performance of the surface. At the beginning of the project, one of the goals was to find origami patterns that maximized surface and volume change, to be applicable for an adaptive façade that could respond various environmental changes.



**Fig. 1.** Experimentation of various origami patterns.

One curiosity and strong possibility for a changing façade was found in the flexagon, an inspiration from Arthur H. Stone's accidental form he stumbled upon while folding his notebook paper at Princeton University in 1939.

*'He had folded the strip diagonally at three places and joined the ends so that it made a hexagon. Then he pinched two adjacent triangles together and pushed the opposite corner of the hexagon toward the center, the hexagon would open out again, like a budding flower, and show a completely new face.'* [10]

Using the changing faces of a similar geometry showed potential for an adaptive façade concept. While Gardner's flexagon discusses a planar version, the authors began to investigate three-dimensional flexagon geometries and their continuous rotational qualities.

### 3.1 Hexa-, Octo-, n- Flexagon Geometry

To gain further understanding of how the flexagon's surfaces are made up, whether from many small joined parts or a single folded surface, multiple material studies were conducted. Figure 2 illustrates a few ways that the surfaces can be made up. The photos from the left show a paper folded hexa-flexagon, followed by 3D printed hexa-, octa-, and deca-flexagons. It may find itself to be solid and continuous or it may only have material in specific areas where the structure is needed. The 3D flexagons symmetrically rotate 360 degrees flipping the faces of each component tetrahedron while changing the opening that lies at the center of the flexagon. As discussed in Elgahzi et al. [8], this property can be advantageously applied for a shading system.



**Fig. 2.** Experimentation of various flexagons.

### 3.2 Rotational Characteristics and Aggregation

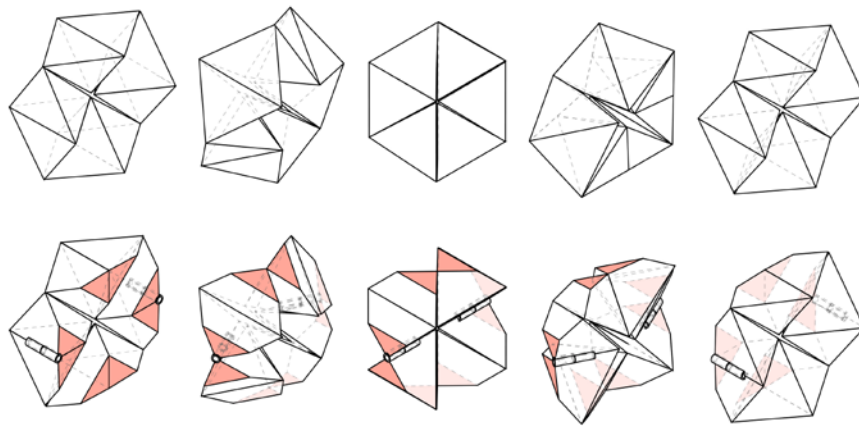
Considering the geometric composition of individual modules Moloney [11] offers a framework for designing patterns of kinetic facades and suggests the visual effect of individual geometry modules (i.e., circular, triangular, rectangular, hexagonal), and argues that hexagonal parts provide the best mix of edge detection and shading depth while allowing compact packing without emphasizing an orthogonal grid. While initially arraying hexa-flexagons modules into larger aggregate surfaces, the authors discovered the difficulties of accommodating rotational mechanisms for staggered hexagonal modules. As an alternative, an octa-flexagon pattern as an orthogonal array was studied (Fig. 3). This pattern allowed to vertically stack multiple units on single shafts for rotational actuation.



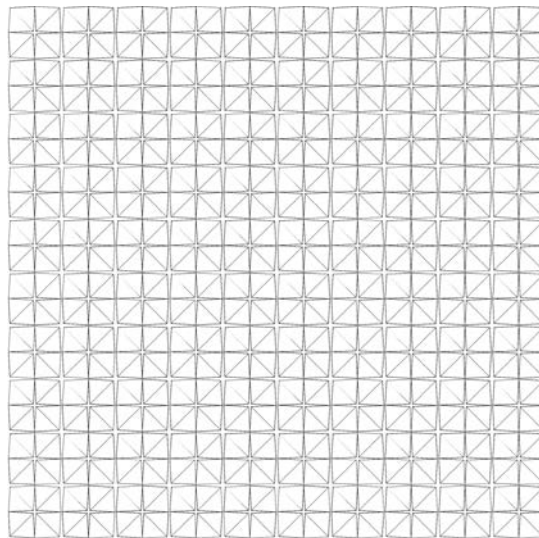
**Fig. 3.** Rotational actuation of octa-flexagon module and front-to-back array with adjacent modules.

One of the design objectives was to achieve symmetrical shaft actuation. When the flexagons were first arrayed using a front to back pattern (right photo in Figure 3) where opening directions alternated for each column. This worked mechanically but as far as performance goes, there was no control over the light levels because each of the columns operated in the exact opposite of the other, negating any benefits or effects caused by any given column.

By Boolean subtracting corners from the octa-flexagon (Fig. 4), the columns no longer needed to be a reaction of one another and could act independently. This independence was gained by the absence of expansion and contraction and the introduction of a set of two drive shafts for each column of flexagons (Fig. 5).



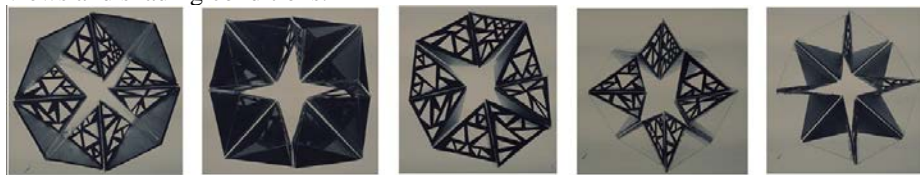
**Fig. 4.** Revised octa-flexagon module (Top row initial module; bottom row after boolean subtraction of corners).



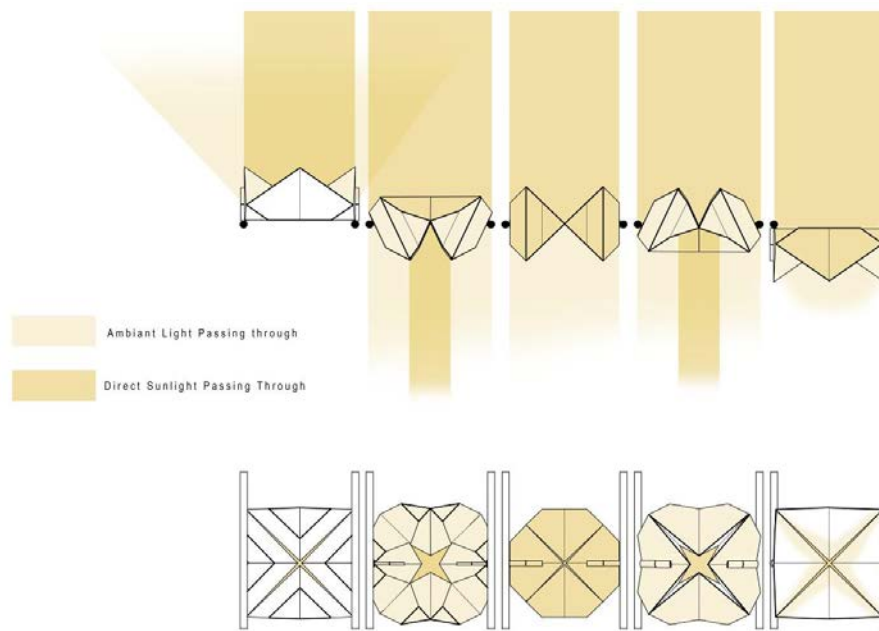
**Fig. 5.** Array of octa-flexagon module as façade screen panel.

### 3.3 Shading and Architectural Qualities

After the octa-flexagon module geometry was finalized, the surface materiality and textures of each tetrahedron surfaces were carefully studied. It was considered to incorporate three types of surface qualities: reflective aluminum, clear glass, and etch patterns. These three surfaces would create a gradient of different shading and views, from clear to opaque, depending on each surface position during the rotation of the tetrahedrons (Fig. 6). Figure 7 diagrammatically explains how the flexagon rotational position, central void opening, and tetrahedron surface finishes position, influence views and shading conditions.



**Fig. 6.** Three types of surface treatments of octa-flexagon tetrahedron modules

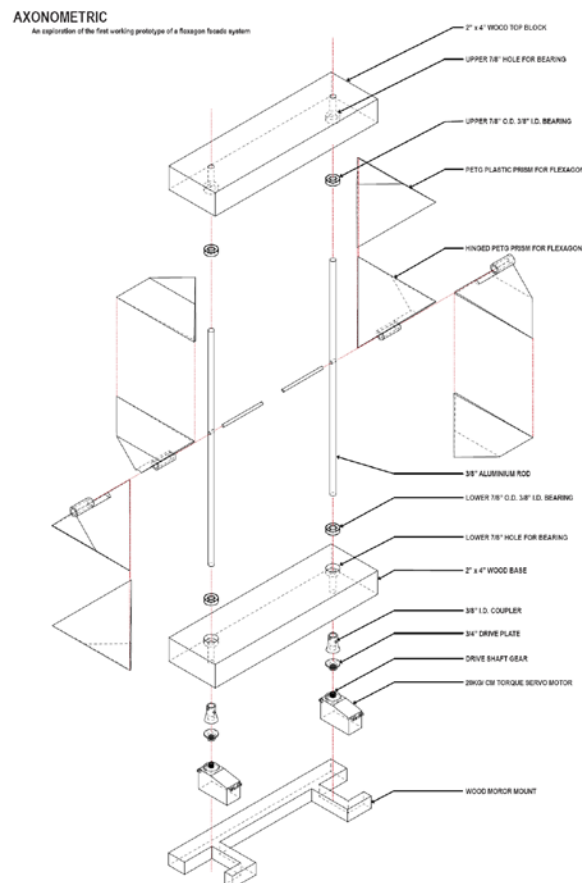


**Fig. 7.** Light and view qualities control through the octa-flexagon modules

### 3.4 Final Mock-up

The mock-up size of the individual octa-flexagon module was determined to be about 1 foot by 1 foot, and to build an array of 5 (horizontal) x 4 (vertical) units (Figure 8). 4 flexagon modules are vertically connected to two shafts that were set in top and bottom bearings of a frame, and in turn actuated by two sets of servo motors. Each flexagon is connected to the shaft through horizontal rods that rotate with the shafts which subsequently flip the flexgons 360 degrees by the rotation of the shafts. The tetrahedron modules surfaces were built from PET-G which had satisfactory transparent qualities while allowing to be laser cut. To emulate the three surface textures and qualities, brushed aluminum sheets were applied for the reflective material, and one of the surfaces were laser scored with a predesigned pattern to create ambient light qualities.

With the use of the Arduino Microcontroller, the prototype included responsive features that would respond to the presence of inhabitant through the use of infrared sensors, and a photocell sensor that responds to changing daylight levels (Fig. 8, Fig. 9).



**Fig. 8.** Mock-up diagram





**Fig. 9.** Full-scale prototype

## 4 Analysis and Discussion

This project aimed to expand the area of responsive shading systems through the novel application of volumetric origami patterns – the flexagon (Fig. 10, Fig. 11). The original contributions of this project came through the design development and prototyping of the kinetics of the flexagon pattern. Departing from relevant research by Elgahzi et al. [8] and Moloney [11], this project identified a specific geometric construct of kinetic actuation of flexagons through vertical shaft actuation, and in addition discovered the performative and aesthetic applicability of surface qualities of the component tetrahedron geometry for daylighting and view control. The aggregation of multiple units also resulted in new understanding of the stacking characteristics and rotational envelope of flexagon geometries. The project outcomes can be summarized as below.

**Aesthetics:** The objective here was to have the greatest amount of surface variation to please the eye with changing materiality and reflection. Another goal was the ability to hide the solid surfaces by using the geometry to orient them in a side view, virtually eliminating their presence on the façade while it is in a clear state. Brushed Aluminum panels were used for their weight performance and thin profile as well as their mat like finish that reflects light without glare hotspots.



**Patterning-** The etch pattern draws light from the center where it is highly concentrated, across the surface it is dispersed then pulled back together before it is dispersed out the edges, creating an ambient glow.

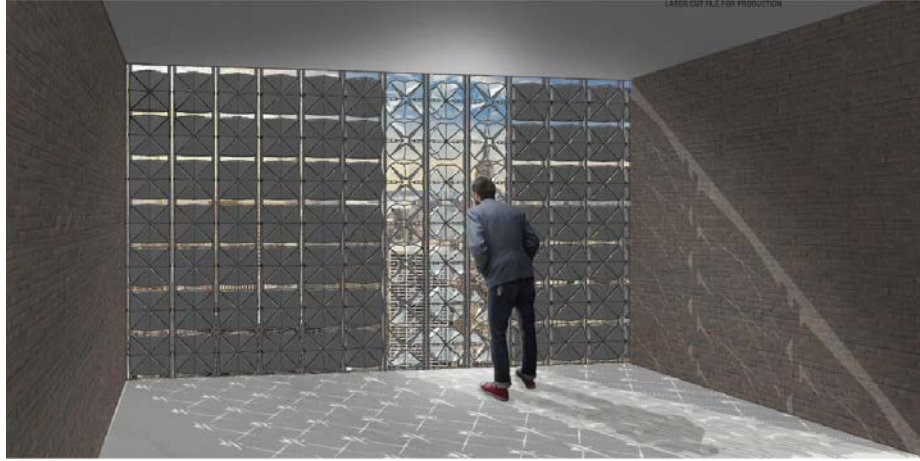
**Daylighting control-** The success of daylight control would be in the ability to have a wide range of shading at different sun angles. This surface is able to present both a solid surface perpendicular to the sun as well as a clear or fritted. This is one of the most beneficial aspects of this façade and the most successful.

**Mechanical-** The ability to move the façade with as little resistance as possible was motivated by the objective to use the smallest size servo as possible, minimizing electrical consumption. The bearings as well as the symmetry were the biggest help for accomplishing this and were very successful in minimizing resistance.

**Fabrication-** The fabrication process was the shortcoming of the project. Although the end prototype worked at a moderate level, there are many improvements that could be made as far as efficiencies in production go, as well as further development to improve construction of modules and their connection to the hinges.



**Fig. 10.** Exterior rendering of proposed flexagon responsive shading system



**Fig. 11.** Interior rendering of proposed flexagon responsive shading system

## 5 Conclusion

Through the design and development of this unique facade, there have been many discoveries that have led to further questions. As a whole, the development has provided a lot of ideas for complex facade systems with simple mechanics. A key accomplishment of this development process was the design of the actuating system. Through the use of high torque and low powered servo motors, in combination with long aluminum rods, this facade was able to create the necessary force required to turn the geometry. Another key factor was the geometry itself and its inherent symmetry and equilibrium which allowed it to rotate with very little resistance. This ability to move with very little resistance made it possible to have an active façade with a high degree of manipulation, without any motors integrated into the surface itself. All of the servo motors were located in the same place at the bottom of the façade making for easy access for maintenance which is a huge accomplishment for a kinetic facade. Some downfalls of the facade are the under development of the face materials and textures of the prisms. In early studies, there was a more in depth investigation of surface texture and material that was lost in translation to the final model. Moving forward with this project, the authors anticipate much more in depth and thorough investigation of light manipulation within the façade including the assessment of the effectiveness of vertically grouped shading units vs. horizontally grouped, or staggered units. Measurable simulations to study the effectiveness for daylighting control are planned.

While the prototype incorporates a simple proof-of-concept responsive algorithm, the code and sensors need to be refined to and be included in the simulation studies to evaluate the effectiveness for daylighting control. Lastly, this design needs to be further developed to evaluate whether how it is to be integrated with contemporary façade systems, including double-skin facades.

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