AN ALGORITHM OF RIGID FOLDABLE TESSELLATION ORIGAMI TO ADAPT TO FREE-FORM SURFACES

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Abstract. When creating new kinds of origami, people design origami creases pattern on 2D plane. Consequently, people unable to precisely envision the 3D folded shape. However, in architecture, civil engineering and industrial applications, an accurate layout is important. This research is to compile an algorithm for creating origami forms with developability and flat-foldability on the target surface, more specifically, by setting a target surface first, generating a Miura-ori tessellation from the geometric configuration of a target surface. We achieve creating origami forms on a target surface, so that we can generate architectural surfaces with folded structure and accurately layout for construction. Our approach facilitates designing a free-form origami structure upon parametric and 3D modelling software for artists, designers and architects.

Keywords. Origami tessellation; free-form; grasshopper3D; rigid foldability; flat-foldability.

1. INTRODUCTION

Various researches have shown that rigid origami has a strong potential for architectural and engineering applications due to geometric flexibility. Tachi, has established mathematical rules of origami and developed software for simulating origami. Examples of software, first, Rigid Origami Simulator for simulating kinematics of rigid origami form (Tachi 2009); second, Free-form Origami to design free-form origami with interactive crease pattern of model by numerous constraints (Tachi 2013). However, the software is not capable of solving the inverse problem of finding the crease pattern that belongs to a given 3D surface. Dudte et al. (2016) employed constrained optimization algorithms to solve the in-verse problem of fitting different curved surface with Miura-ori tessellation. While Tsiamis (2018)explores origami tessellation to adapt to free-form surface on parametric and 3D modelling software and analyzes the performance of origami structure. The research summarized in this paper aims to build upon the work of Dudte and Tsiamis by studying how Miura-ori tessellation fit to target surface with various curvature.

The objective of this research is to explore the ability of folded Miura tessellations with rigid foldability and flat-foldability, made from tiling free-form

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surface configurations with Miura-ori unit modules. The method is tested on Rhinoceros and Grasshopper. Grasshopper 3D is a graphical algorithm editor tightly integrated with Rhinoceros, provides a popular design environment for parametric design and active community. The first stage of the research is to simulate the geometry and kinematics of origami module of Miura-ori in order to make a Miura tessellation. The second stage is to compile algorithms to generate a Miura-ori tessellation to fit surfaces with specific curvature. In order to attain the folded Miura-ori tessellation, Kangaroo, a plugin of grasshopper with physical engine, has to balance the effects of multiple rules, including the bending force, equalization, mathematical methods to set each angle between mountain and valley patterns, planarization of mesh faces and the "PullToSurf" which forces the vertices of Miura tessellation to move under constraint on target surface. Parametric modelling techniques aid to deform the digital representations of the folded Miura tessellation to match the curvature of various surfaces. The final stage is to materialize the outcomes, to confirm the feasibility of the algorithm, I use sheets of laser engraving paper with origami pattern and fold them manually on two types of mathematical surface, the doubly curved paraboloid and the hyperbolic paraboloid. Finally, according to result of feasibility of our algorithm, we generate a Miura-ori tessellation on a free-form surface.

2. PARAMETERIZATION AND KINEMATICS OF THE ORIGAMI UNIT MODULE

Origami tessellation composes of identical unit modules, shaped from simple patterns. Geometry of Miura pattern can be parameterized in various ways. we view all the creases of Miura-ori pattern as two kinds of lines, horizontal and zigzag. All the vertices of zigzag lines and intersection of the two types of lines can form parallelograms, arranged in rows and columns, thus, a Miura-ori pattern. By parameterizing the relation of two kinds of lines, we can adjust the size and amounts of each parallelogram.

2.1. MIURA-ORI UNIT MODULE

Figure 1 shows that the defining process of Miura-ori unit module. First, we define the length of line AC which is divided into 2 lines with equal length segment. The point B are defined by segment AB and BC. Second, line DF and line GI are defined by the translation of the line AC along the Y vector. Especially, line DF provide the ability of movement along the X vector. The 4 meshes of parallelograms can be composed of face ABED, face BCFE, face FIHE and face EHGD. We also can adjust the scale of Miura unit by changing vector Y of horizontal lines and moving mid-line which is line DF along vector X.

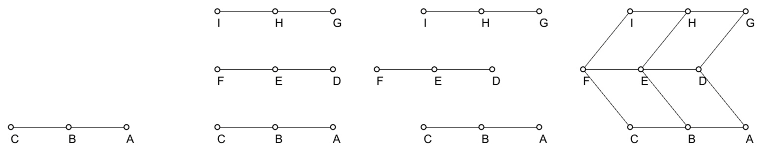

Figure 1. The steps of developing a Miura-ori unit module from vertices and segments.

2.2. MIURA-ORI TESSELLATION

We define the Miura unit module this way; furthermore, it can be developed into a pattern of Miura-ori tessellation. As one can see, we can increase length of line AC and divide line AC into more segments. Each four end points of the segments, according to the order that we set, can form a parallelogram, which in turn, can form the Miura-ori tessellation.

2.3. KINEMATIC SIMULATION

This simulation differs from the common method widely used, in kangaroo components, “Origami” and “Hinge”. We choose the method of setting loads to pull the mesh of tessellation by kangaroo components, including “AnchorXYZ”, “Load” and “Lenghth”.With the knowledge of the kinematic of the unit module, the development of Miura-ori tessellation (Figure 2) is able to be simulated; consequently, we will have digital models of Miura-ori tessellation to foresee any obstacles we might encounter in reality.

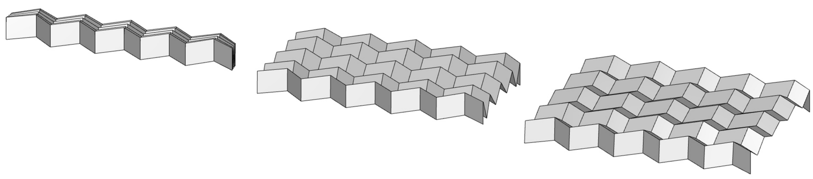

Figure 2. kinematic simulation of Miura-ori tessellation.

3. ALGORITHMS TO GENERATE A RIGID FOLDABLE MIURA-ORI TESSELLATION TO FIT SURFACES

The algorithms developed within the Grasshopper are visual programming environment. Using the plugin, Kangaroo, to set constraints (rigid-foldability, flat-foldability, etc.). In order to parametrically generating Miura-ori tessellation to adapt to the target surface, we convert the several constraints, including developability, flat-foldability and planarity, into physical method and existing components of kangaroo through defining the combination of components in grasshopper.

3.1. GENERATE MIURA-ORI TESSELLATION ON THE TARGET SURFACE

Miura-ori tessellation is rigidly foldable, all of parallelogram panels would be able to fold continuously from a flat to a fully fold state without deformation of the panels. In other words, Miura-ori tessellation pattern has properties of developability and flat-foldability. In order to better preserve developability on kangaroo, we arranged Miura-ori tessellation pattern on the target surface (Figure 3), by means of "Divide Surface", a component in Grasshopper. It can generate a grid of points on the target surface. The parameters of UV influence the number of Miura unit pattern.

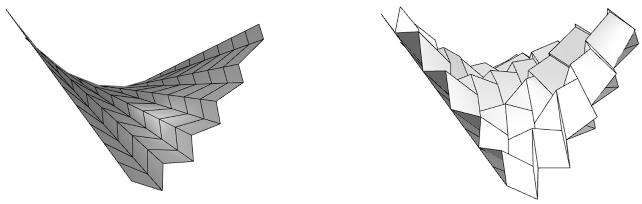

Figure 3. Generation of pattern of Miura-ori tessellation on targert surface (hyperbolic paraboloid), simulation with all constraints in this research.

3.2. DEVELOPABILITY CONSTRAINTS

According to Kawasaki's Theorem, we understand that crease patterns with each vertex that may be unfolded back to a flat figure. In developability constraints, we know that:

$$\alpha 1 + \alpha 2 + \alpha 3 + \alpha 4 = 2\pi \quad (1)$$

In this condition, we can achieve it by using the component called "Developablize" in Kangaroo. It enables the whole folded Miura-ori tessellation to be a flat sheet because the sum of the neighbouring sector angles is 2π. The one of the most salient effect is to the combinations of "Developablize" and "PlanarizeQuads"(Figure 4). By doing this, it make meshes have characteristic of paper sheets.

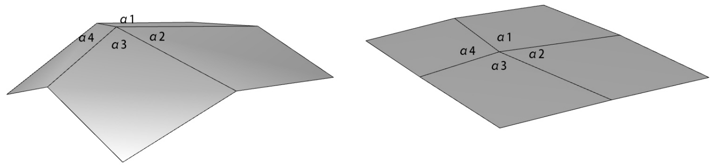

Figure 4. Under constraints of planarity and developabilty to simulate, the left mesh-es enable transform to right meshes. Specially, Each neighbouring mesh of a vertex can be coplanar. The sum of the neighbouring sector angles is less than 2π(left). The sum of the neighbouring sector angles is 2π(right).

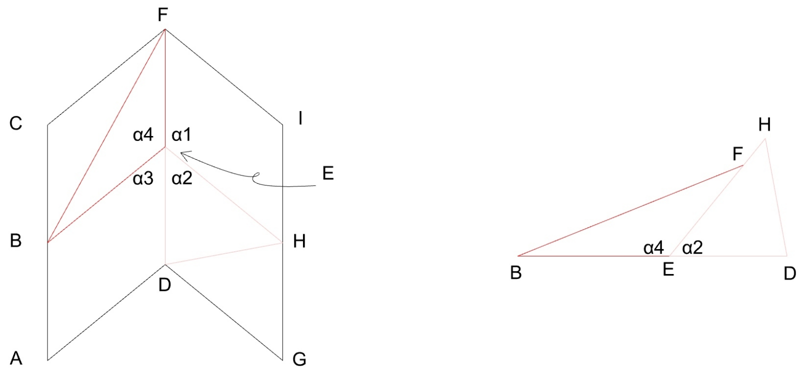

Figure 5. In base faces of Miura-ori unit module (left), singling out geometry(right) of triangle BEF (red) and triangle EDH (pink) for applying bend and equalize forces.

3.3. PLANARITY CONSTRAINTS

Planarity makes sure that every vertex in each mesh of parallelogram in folded origami is coplanar. Under this condition, we can achieve it by using a component called "PlanarizeQuads" in Kangaroo.

3.4. FLAT-FOLDABILITY CONSTRAINTS

Since we do the procedure inversely, compared to the conventional method, we need to preserve the Flat-foldability, the sum of each pair of opposite interior angles around each vertex must equal to π. In each vertex of Generalized Miura tessellation pattern should align with the following two equations:

$$\alpha 1 + \alpha 3 = \alpha 2 + \alpha 4 \tag{2}$$

$$\alpha 1 + \alpha 3 = \pi \tag{3}$$

we attempted to achieve the aforementioned equations by using Kangaroo engine based on physics. In this part, the two main forces are bending force and equalization in Kangaroo. First, by singling out the same triangle BEF and EDH from the Miura unit, we can simulate it with Miura-ori tessellation in the meantime. In addition, Based on SSS theorem of triangle, relation of α2 and α4 can be simulated. Bending forces enable line BE and line ED to be collinear, so it means that the sum of the radian of α2 and α4 equals π. Equalizing lines in two parts which are left and right (Figure 5), it can force corresponding lines to always have the same edge lengths. If it has the same lengths in two parts, α2 and α4 in Miura unit constantly sum up as π.

3.5. IMPLEMENTATION OF SIMULATION ON TARGET SURFACE

In order to generate a folded origami to fit the target surface, the various effects of forces in Kangaroo (Figure 6) have to be balanced. The forces, “SpringsFromLines”, “Bend”, “Equalize”, “Developablize” , “PullToSurf” and “PlanarizeQuads” constitute different constraints respectively. In this part, the most vital force is “PullToSurf” because, after duplicating the same surface on top of the target surface, we are able to pull each points E of Miura units module to the duplicated one, to form folded Miura-ori tessellation shape (Figure 7). Moreover, points ACGI in Mirua unit are anchored on the target surface, as a result, they are unmoved throughout simulation.

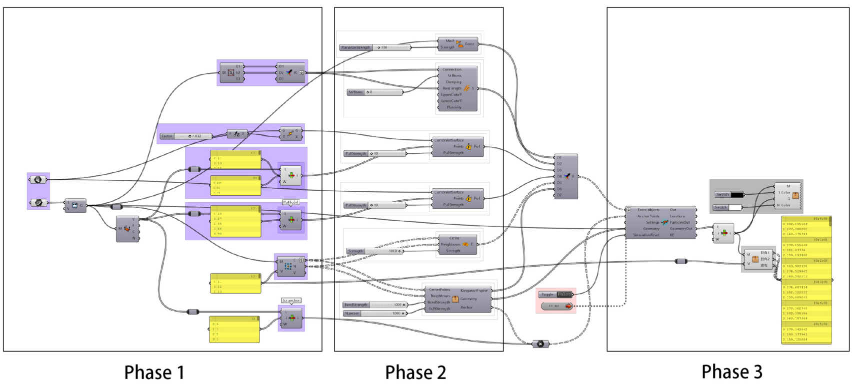

Figure 6. The proposed Grasshopper definition to simulate using kangaroo, phase 1 for process of mesh and input of both target surface and Miura pattern, phase 2 for combination of multiple forces, phase 3 for Kangaroo engine and output of results.

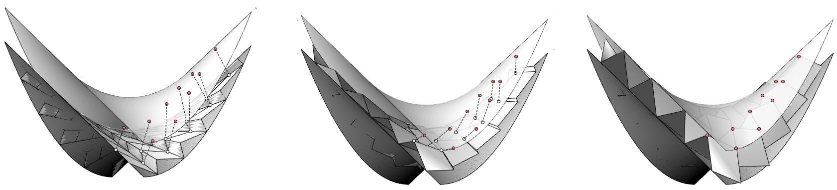

Figure 7. During the iteration of Kangaroo, each point E (white points) of Miura-ori unit is attracted to each corresponding points(red points) on duplicated sur-faces by force "PullToSurf".

This assures that, during the simulation, folded Miura-ori tessellation closely attaches to the target surface.During the process of the folding simulation, yielding a folded geometry state with all the constraints requires applying the strengths with each of these different forces to the mesh model.

4. PAPER MODEL EXPERIMENT

The final stage is to realize the result, we test on two types of mathematical surface, the doubly curved paraboloid (Figure 8) and the hyperbolic paraboloid (Figure 9), and then use bristol board of laser engraving paper with flat crease pattern of tessellation and fold them manually to testify the feasibility of the algorithm. By cause of the state of simulation outcome being folded, we unfold folded origami to flat state by using Free-form Origami for its facilitation of the process of unfolding origami and we obtain crease pattern of simulation in flat state more easily. Then, using Rhino, we differentiate the mountain creases from the valley creases to be solid and dashed lines respectively for we need different power to laser-cut out the mountain and valley creases. After laser-cutting the tessellations onto a sheet of paper, we fold these structures and attach the four vertices in each unit to the corresponding points on the target surface.

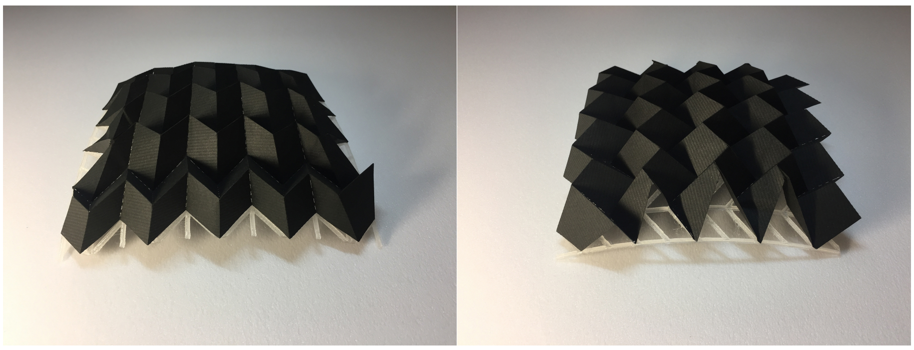

Figure 8. Elliptic paraboloid - positive Gauss curvature.

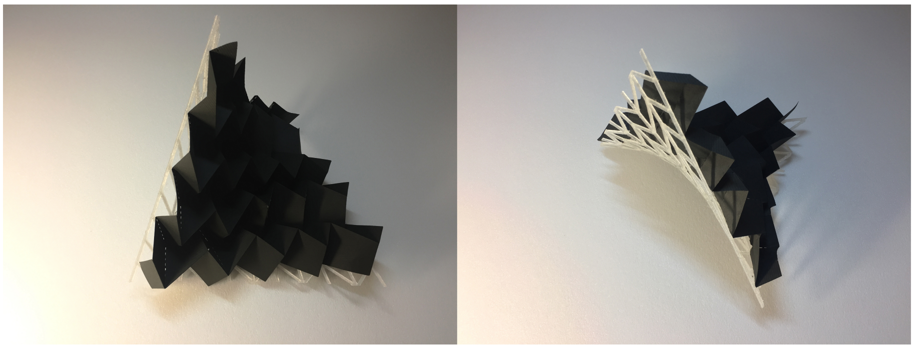

Figure 9. Hyperbolic paraboloid - negative Gauss curvature.

5. RESULT OF SIMULATION ON FREE-FORM SURFACE

We define the free-form surface based on our paper model experiment . The positive and negative Gauss curvature can coexist on free-form surface. Figure 10 shows an example of this testing. We simulate the Miura-ori tessellation on the target surface of free-form with all constraints we mentioned in this research.

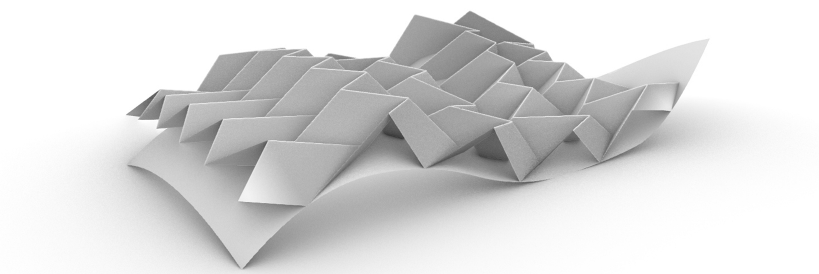

Figure 10. Implementation on a free-form surface.

6. FURTHER WORK

The method presented in this paper may contributes to applications in art, design and engineering. By manipulating the parameters of scale and amount of Miura-ori unit module, it is beneficial to apply origami into various fields for manufacture. We discovered that if the length of a studied object is too long, edges of its mesh would collapse. This may be solved by controlling the limitation of length of each edges. In addition, each angle in Mirua-ori tessellation unit can approach accurately to the values we got from our constraints converted from mathematical rules.

7. CONCLUSION

We present our goal and approach with sufficient details for replication so researchers can follow the method we define in grasshopper to generate Miura-ori tessellation pattern. We also presented a physical model to confirm feasibility of algorithms. In this research, algorithms compiled upon grasshopper, integrated the generation of geometric models and ability of parameterization of Miura tessellation, and then simulate fitting the target surface to be folded origami state with several constraints, by using Kangaroo Plugin. The result of simulation, in Miura-ori unit, most of the opposite angles approximate to π, all sum of the neighbourhood sector angles closely approximate to 2π (Figure 11). The angle is unable to align with the constraints due to structural performance and natural physical response of Miura-ori tessellation.

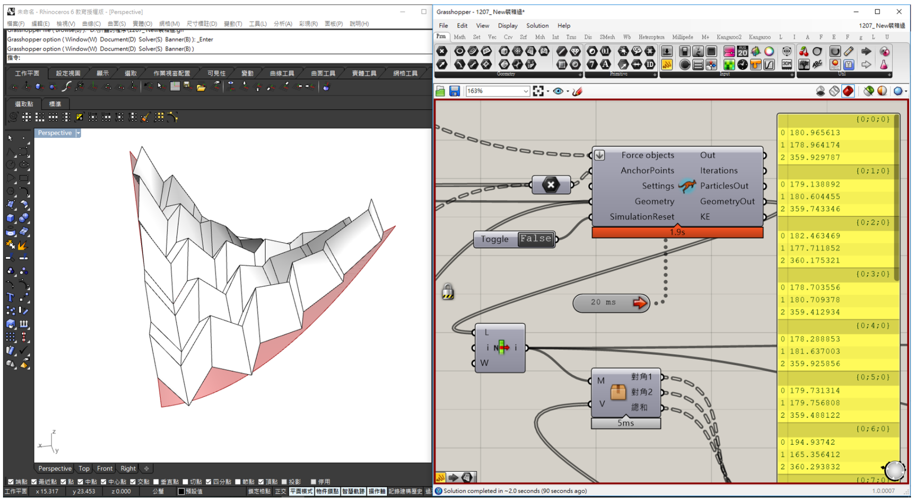

Figure 11. During the simulation of origami(left), the yellow panel (right) shows that information of angles in each Miura unit module. Both 0 and 1 are sum of opposite angles; 2 is sum of the neighbourhood sector angles.

Distortion of crease pattern in tessellation, like the Miura-ori tessellation, to fit a target surface with specific curvature, gives origami tessellations great possibility to design target geometry at different scales in context with differing field criteria. Finally, we see potential in exploring the design possibilities of origami structure, due to the fact that all vertices are controlled and attached on both surfaces, surfaces can be adapted easily on the folded Miura-ori tessellation. By structure performance of tessellation, it is economical to use paper to produce thin surface with various curvatures.

8. Citations and References

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