Modeling of bending-torsion couplings in active-bending structures. Application to the design of elastic gridshells.

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1 author:

Lionel du Peloux
École des Ponts ParisTech

ELASTIC GRIDSHELL

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in active-bending structures

application to

THE DESIGN OF ELASTIC GRIDSHELLS

Lionel du Peloux de Saint Romain
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THESE DE DOCTORAT

Modeling of bending-torsion couplings in active-bending structures. Application to the design of elastic gridshells.

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Auteur	Lionel **DU PELOUX DE SAINT ROMAIN**
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Président	Bernard **MAURIN**
Rapporteurs	Sébastien **NEUKIRSH**
	Carlos **LÁZARO**
Examinateurs	Alberto **PUGNALE**
	Jean-François **CARON**
	Cyril **DOUTHE**
Invité	Bernard **VAUDEVILLE**
Directeur	Olivier **BAVEREL**

“*Quia nominor leo.*”

À Jacques & Christiane,
Mes grands-parents bien-aimés.

ABSTRACT

An *elastic gridshell* is a freeform structure, generally doubly curved, but formed out through the reversible deformation of a regular and initially flat structural grid. Building curved shapes that may seems to offer the best of both worlds : shell structures are amongst the most performant mechanically speaking while planar and orthogonal constructions are much more efficient and economic to produce than curved ones. This ability to "form a form" efficiently is of peculiar importance in the current context where morphology is a predominant component of modern architecture, and envelopes appear to be the neuralgic point for building performances.

The concept was invented by Frei Otto, a German architect and structural engineer who devoted many years of research to gridshells. In 1975 he designed the Multihalle of Mannheim, a 7500 m^2 wooden shell which demonstrated the feasibility of this technology and made it famous to a wide audience. However, despite their potential, very few projects of this kind were built after this major realization. And for good reason, the resources committed at that time cannot guarantee the replicability of this experiment

for more standard projects, especially on the economic level. Moreover, the technics and methods developed by Otto's team in the 1960s have mostly fall into disuse or are based on disciplines that have considerably evolved. New materials, such as composite materials, have recently emerged. They go beyond the limitations of conventional materials such as timber and offer at all levels much better technical performances for this kind of application. Finally, it should be noted that the regulatory framework has also deeply changed, bringing a certain rigidity to the penetration of innovations in the building industry. Therefore, the design of gridshells arises in new terms for current architects and engineers and comes up against the inadequacy of existing tools and methods.
In this thesis, which marks an important step in a personal research adventure initiated in 2010, we try to embrace the issue of the design of elastic gridshells in all its complexity, addressing both theoretical, technical and constructive aspects. In a first part, we deliver a thorough review of this topic and we present in detail one of our main achievements, the ephemeral cathedral of Créteil, built in 2013 and still in service. In a second part, we develop an original discrete beam element with a minimal number of degrees of freedom adapted to the modeling of bending and torsion inside gridshell members with anisotropic cross-section. Enriched with a ghost node, it allows to model more accurately physical phenomena that occur at connections or at supports. Its numerical implementation is presented and validated through several test cases. Although this element has been developed specifically for the study of elastic gridshells, it can advantageously be used in any type of problem where the need for an interactive computation with elastic rods taking into account flexion-torsion couplings is required.

Keywords : gridshell, form-finding, active-bending, free-form, torsion, elastic rod, coupling, fibreglass, composite material.

RÉSUMÉ

Les structures de type *gridshell élastique* permettent de réaliser des enveloppes courbes par la déformation réversible d'une grille structurelle régulière initialement plane. Cette capacité à "former la forme" de façon efficiente prend tout son sens dans le contexte actuel où, d'une part la forme s'impose comme une composante prédominante de l'architecture moderne, et d'autre part l'enveloppe s'affirme comme le lieu névralgique de la performance des bâtiments.
Fruit des recherches de l'architecte et ingénieur allemand Frei Otto dans les années 1960, elles ont été rendues populaires par la construction de la Multihalle de Mannheim en 1975. Cependant, en dépit de leur potentiel, très peu de projets de ce type ont vu le jour suite à cette réalisation emblématique qui en a pourtant démontré la faisabilité à grande échelle. Et pour cause, les moyens engagés à l'époque ne sauraient assurer la reproductibilité de cette expérience dans un contexte plus classique de projet, notamment sur le plan économique. Par ailleurs, les techniques et les méthodes développées alors sont pour la plus part tombées en désuétude ou reposent sur des disciplines scientifiques qui ont considéra-

blement évoluées. Des matériaux nouveaux, composites, ont vu le jour. Ils repoussent les limitations intrinsèques des matériaux usuels tel que le bois et offrent des performances techniques bien plus intéressantes pour ce type d'application. Enfin, notons que le cadre réglementaire a lui aussi profondément muté, apportant une certaine rigidité vis-à-vis de la pénétration des innovations. Ainsi la conception des gridshells se pose-t-elle en des termes nouveaux aux architectes et ingénieurs actuels et se heurte à l'inadéquation des outils et méthodes existant.

Dans cette thèse, qui marque une étape importante dans une aventure de recherche personnelle initiée en 2010, nous tentons d'embrasser la question de la conception des gridshells élastiques dans toute sa complexité, en abordant aussi bien les aspects théoriques que techniques et constructifs. Dans une première partie, nous livrons une revue approfondie de cette thématique et nous présentons de façon détaillée l'une de nos principales réalisation, la cathédrale éphémère de Créteil, construite en 2013 et toujours en service. Dans une seconde partie, nous développons un élément de poutre discret original avec un nombre minimal de degrés de liberté adapté à la modélisation de la flexion et de la torsion dans les gridshells constitués de poutres de section anisotrope. Enrichi d'un noeud fantôme, il permet de modéliser plus finement les phénomènes physiques au niveau des connexions et des appuis. Son implémentation numérique est présentée et validée sur quelques cas tests. Bien que cet élément ait été développé spécifiquement pour l'étude des gridshells élastiques, il pourra avantageusement être utilisé dans tout type de problème où la nécessité d'un calcul interactif avec des tiges élastiques prenant en compte les couplages flexion-torsion s'avère nécessaire.

Keywords : gridshell, form-finding, active-bending, free-form, torsion, elastic rod, coupling, fibreglass, composite material.

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INTRODUCTION

La paternité des structures de type *gridshell élastique* est couramment attribuée à l'architecte et ingénieur allemand Frei Otto, qui les a intensivement étudiées au XX$^{\text{ème}}$ siècle. Fruit de son travail de recherche, il réalise en 1975, en collaboration avec l'ingénieur Edmund Happold du bureau Arup, un projet expérimental de grande ampleur : la Multihalle de Mannheim [1, 2]. Cette réalisation emblématique ancrera durablement les gridshells dans le paysage des typologies structurelles candidates à l'avènement de géométries non-standard, caractérisées par l'absence d'orthogonalité. Cette capacité à *former la forme* de façon efficiente prend tout son sens dans le contexte actuel où, d'une part la forme s'impose comme une composante prédominante de l'architecture moderne (F. Gehry, Z. Hadid, ...) et d'autre part l'enveloppe s'affirme comme le lieu névralgique de la performance des bâtiments, notamment environnementale.

Littéralement, le terme *grid-shell* désigne une résille à double courbure dont le comportement mécanique s'apparente à celui d'une coque ; c'est à dire que les efforts y transitent principalement de manière membranaire. Ces ouvrages peuvent franchir de grandes portées en utilisant un minimum de matière. Cependant, il semble plus rigoureux et plus fidèle à l'histoire de désigner par *gridshell élastique* la combinaison indissociable d'un principe structurel – le gridshell, une résille qui fonctionne telle une coque – et d'une méthode constructive astucieuse – la déformation réversible d'une grille de poutre initialement plane pour former une surface tridimensionnelle à double courbure. Le projet de Mannheim – dans lequel une grille en bois de trame régulière, initialement plane et sans rigidité de cisaillement est déformée élastiquement jusqu'à la forme désirée via un dispositif d'étaiement, puis contreventée pour mobiliser la raideur d'une coque et finalement couverte d'une toile – pose les bases de ce nouveau concept et le rend populaire auprès d'un large public d'architectes et d'ingénieurs de par le monde.

Cependant, en dépit du potentiel de cette typologie, très peu de projets ont vu le jour suite à la construction de la Multihalle. Il faut en effet attendre 25 ans et le développement des méthodes de calcul numérique pour voir de nouveau éclore quelques réalisations iconiques : Shigeru Ban innove en passant du bois au carton pour la construction du Pavillon de Hanovre en 2000 [3] ; puis viennent les gridshells en bois de Downland en 2002 [4] et de Savill en 2006 [5] qui reprennent fidèlement les principes développés à Mannheim mais emploient des méthodes constructives différentes. Depuis une dizaine d'années le laboratoire Navier a investi ce champ de recherche sous le double aspect de la structure et du matériau, donnant lieu à la réalisation de quelques prototypes (en 2006 et 2007 [6, 7]) et des deux premiers bâtiments de type gridshell élastique en matériau composite construits à ce jour (Solidays 2011 [8] et Créteil 2013 [9]).[1] Plus récemment, on a pu observer un certain engouement pour la construction de pavillons en bois de petite taille, non couverts, réalisés selon des principes similaires à ceux de la Multihalle, essentiellement dans le cadre de workshops pédagogiques ou bien de projets de recherche [10, 11, 12, 13].

Il est naturel de se demander pourquoi cette innovation prometteuse peine ainsi à essaimer ? S'il est vrai que la construction de la Multihalle de Mannheim a permis de prouver la faisabilité économique et technique du concept de gridshell élastique à grande échelle, il faut bien reconnaître que cette prouesse n'a été rendue possible qu'au terme d'un long processus de maturation pour développer et acquérir l'ensemble des compétences scientifiques, techniques, méthodologiques et humaines nécessaires à sa conception et à sa construction.[2]

1. Ici, le matériau employé, un composite à base de fibres de verre imprégnées dans une matrice polyester et obtenu par pultrusion, apporte un gain de performance très significatif par rapport au bois et permet de rester sur une conception à simple nape là où le bois aurait nécessité une grille à double nape beaucoup plus complexe à réaliser.

2. "This is not a case of a building creatively designed, but based on a support system of additive known elements. This design is the result of a symposium of creative thought in the formation, the invention of building elements with the

En vérité, une telle dépense de moyens pour développer et rassembler ces compétences ne saurait assurer la reproductibilité de cette experience sauf en de très rares occasions et pour des projets d'exception. Par ailleurs, les techniques développées à l'époque sont pour partie tombées en désuétude (e.g. la recherche de forme par maquette physique) ou bien ont fortement évoluées voir même mutées (e.g. le calcul numérique). Des matériaux nouveaux, composites, ont vu le jour. Ils repoussent les limitations intrinsèques des matériaux usuels tel que le bois et offrent des performances techniques bien plus intéressantes pour ce type d'application (durabilité, allongement à la rupture, légèreté, résistance mécanique, fiabilité de niveau industrielle, ...). Enfin, notons que le cadre réglementaire s'est considérablement étoffé apportant aussi son lot de rigidités vis-à-vis de la pénétration des innovations dans le secteur de la construction.

Ainsi la conception des gridshells se pose-t-elle en des termes nouveaux aux architectes et ingénieurs actuels. Elle se heurte aux deux difficultés majeures suivantes :

- La première difficulté est d'ordre technique et concerne la fonctionnalisation de la structure. En effet, bien que le principe du gridshell permette de réaliser des ossatures courbes de manière optimisée, il n'en reste pas moins complexe de constituer à partir de cette résille porteuse une véritable enveloppe de bâtiment capable de répondre à un large panel de critères performantiels (tels que l'étanchéité, l'isolation thermique, l'isolation acoustique, ...) sur un support qui ne présente aucune rationalité géométrique.[3]

- La seconde difficulté est d'ordre théorique et concerne la mise au point d'outils et de processus de conception adaptés à l'étude de ces structures d'un genre nouveau où Architecture et Ingénierie collaborent de manière indissociable à l'identité formelle de l'ouvrage. L'inadéquation des méthodes et des outils de design actuels, orientés davantage vers la justification des ouvrages que vers leur conception, constitue un des principaux freins à la diffusion de cette innovation.

Le présent manuscrit s'articule autour de deux grandes parties qui tentent chacune de construire des éléments de réponse aux défis identifiés précédemment. La première partie, composée des chapitres 1 et 2, est destinée à présenter en profondeur le concept de gridshell

simultaneous integration of the theoretical, scientific contributions from mathematics, geodesy, model measuring, statics as well as control loading and calculation. We are dealing with more than pure 'teamwork', we are dealing with team creation." [Georg Lewenton 1, p. 201]

3. Pour contourner cette difficulté, une approche prometteuse consiste à identifier des classes de surfaces courbes (comme les maillages isoradiaux) dont certaines propriétés géométriques (e.g. facettes planes, noeuds sans torsion) s'avèrent avantageuses sur le plan constructif [14].

élastique, son potentiel et les difficultés techniques sous-jacentes (voir partie I). La seconde partie, composée des chapitres 3 à 6, est consacrée au développement d'un élément de poutre discret original prenant en compte les sollicitations de flexion et de torsion et applicable à tout type de section dont le centre de torsion est confondu avec le centre de masse, ainsi que certains types de discontinuités liées à la présence de connexions dans les résilles de type gridshell (voir partie II). Cette seconde partie constitue le coeur *académique* de ce travail de thèse.

Dans le chapitre 1 nous rappelons la genèse de cette invention et nous en donnons une définition précise et actualisée. Puis nous dressons un état des lieux critique des projets réalisés sur ce principe depuis le début des années 1960 à nos jours. Cette brève histoire des gridshells dessine à elle seule le potentiel de ces structures, notamment en terme d'expression formelle et de performance structurelle. Loin de les enfermer dans un style d'architecture particulier, elle en souligne au contraire la formidable variété. Cette revue de projet est complétée par une revue approfondie de la littérature existante sur l'ensemble des domaines connexes à cette thématique (géométrie, structure, matériaux, logiciel).

Dans le chapitre 2 nous présentons de manière détaillée la conception et la réalisation de la cathédrale éphémère de Créteil, un gridshell élastique en matériau composite construit en 2013 et toujours en service. Cette expérience peu commune a été une source inépuisable pour alimenter ce travail de thèse. Cette relecture expose les méthodes et les outils de conceptions développés pour faire aboutir le projet, les difficultés rencontrées, les pistes d'amélioration. Elle fournit également une analyse économique pour cerner les axes de progrès prioritaires dans l'optique d'une commercialisation future.

Dans le chapitre 3 nous rappelons les notions fondamentales déjà connues, indispensables à notre étude, pour la caractérisation géométrique de courbes de l'espace et de repères mobiles attachés à des courbes. Ces notions sont présentées pour le cas continu puis pour le cas discret ; ce dernier étant essentiel pour la résolution numérique de notre modèle. Cependant, nous observons que la notion clef de courbure géométrique perd son univocité dans le cas discret. Nous identifions alors plusieurs définitions de la courbure discrète. Puis nous les comparons selon des critères propres à notre application (convergence géométrique, représentativité énergétique, forme d'interpolation). A l'issu de cette analyse, la définition la plus pertinente est retenue pour le développement du nouveau modèle numérique au cours des chapitres suivants.

Dans le chapitre 4 nous élaborons un premier modèle de poutre à 4-DOFs par une approche variationnelle. Ici nous reprenons et enrichissons un travail initié lors d'une précédente thèse [15] inspirée par des travaux récents sur la simulation des tiges élastiques dans le domaine des *computer graphics* [16], et à laquelle j'ai collaboré [17, 18]. En particulier, notre développement permet d'aboutir à des expressions purement locales des efforts internes et prouve l'équivalence avec le membre statique des équations de Kirchhoff. Sur le plan mathématique, le modèle est développé en continu et son implémentation numérique n'est pas traité.

Dans le chapitre 5 nous développons une nouvelle approche, plus directe et plus complète, pour construire à partir des équations de Kirchhoff un élément de poutre enrichi par un noeud fantôme et possédant lui aussi un nombre de degré de liberté minimal. L'originalité de cet élément est de pouvoir localiser proprement dans l'espace certains types de discontinuités, notamment des discontinuités de courbures provoquées par des efforts ponctuels ou des sauts de propriétés matérielles. Cela permet une modélisation plus fine des phénomènes physiques au sein de la grille, aussi bien au niveau des connexions que des conditions aux appuis, ce qui était le principal objectif de ce travail de thèse.

Dans le chapitre 6 nous combinons les résultats des chapitres précédents pour construire un élément de poutre discret tout à fait adapté à la modélisation numérique des gridshells élastiques. Nous présentons la construction de cet élément et la méthode de résolution numérique employée pour trouver l'état d'équilibre statique du système, à savoir le relaxation dynamique. Enfin, nous donnons quelques éléments sur *Marsupilami*, le programme informatique que nous avons mis au point et qui implémente l'élément de poutre discret élaboré au cours de cette thèse. Nous exposons aussi quelques résultats de comparaison avec des logiciels du commerce qui ont permi de valider notre travail. Plus généralement, l'élément développé convient bien pour modéliser des problèmes de couplage flexion-torsion dans des poutres élancées, comme par exemple les phénomènes de repositionnement des câbles et des gaînes accrochées aux bras robots, un matériel industriel qui se démocratise à grande vitesse.

Part I

ELASTIC

GRIDSHELL

Chapter 1
ELASTIC GRIDSHELLS

1.1 INTRODUCTION

This chapter is meant to define and introduce what elastic gridshell structures are. It develops a comprehensive but precise view of the numerous knowledge and know-how that gravitate around this concept.

1.1.1 OVERVIEW

We naturally begin this chapter by defining the notion of elastic gridshell and the context in which this technology arose (see §1.2). We briefly highlight the benefits of composite materials for this kind of structure. We then propose two thorough reviews : the first one is dedicated to known built elastic gridshell structures (see §1.3) while the second one is a literature review of the main works related to the topic of elastic gridshells (see §1.4).

1.1.2 CONTRIBUTIONS

- We establish a chronological review of known built elastic gridshells, from the very beginning of this technology to the present time. We reveal the richness of this concept by exhibiting the great variety of realised projects. We discuss the specificities brought by each one of these projects.

- We establish an up-to-date review of the existing scientific literature, crossing multiple fields of research (geometry, mechanics, material, ...).

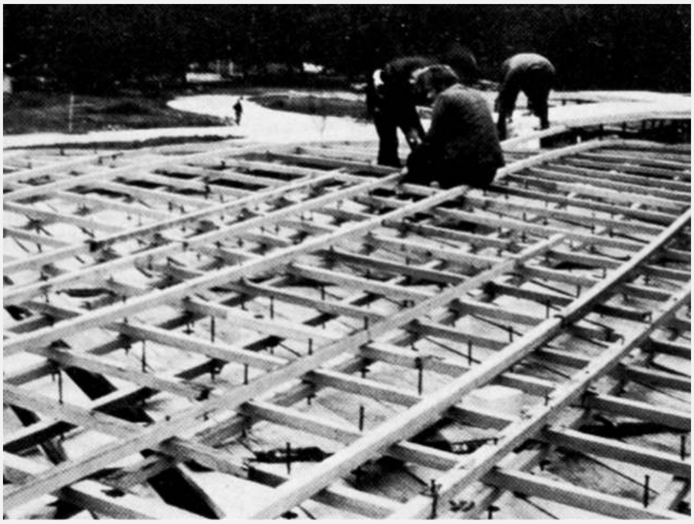
1.2 DEFINITION

The invention of the *elastic gridshell* concept is commonly attributed to Frei Otto, a German architect who devoted several years to gridshells. In 1975 he achieved the famous *Mannheim Multihalle* [2], a wooden shell of 7500 m^2, in collaboration with the engineer Edmund Happold (Arup). Literally, the word “gridshell” refers to grids behaving like shells : from a mechanical point of view that means stresses acting on the structure are mainly transmitted through compression and tension. These structures can cross large-span with very little material.
However, according to the historic evolution of the concept, to characterise a gridshell as the combination of a structural concept (a grid behaving like a shell, see §1.2.2) and a specific construction process (see §1.2.1) using the bending flexibility of the material (see §1.2.3) seems to be more accurate. The project of Mannheim – in which a wooden regular and planar grid, lacking shear stiffness, is elastically deformed up to a targeted shape with the help of stays, and then braced and covered – is regarded as the starting point of this new concept (see figs. 1.1a to 1.1d).
This type of gridshell, known as elastic gridshell, offers a very elegant manner to materialise freeform shapes from an initially flat and regular grid, which obviously has many practical benefits : planar initial geometry, standard connection nodes, standard profiles and so on. Note that the term *rigid gridshell* is often opposed to the term *elastic gridshell* to indicate reticulated structures that behave like shells but are not formed in an active-bending process.

1.2.1 ERECTION PROCESS

Usually, the grid morphology is not trivial and leads to design numerous costly and complex joints. To overcome this issue, an original and innovative erection process was developed

1.1a



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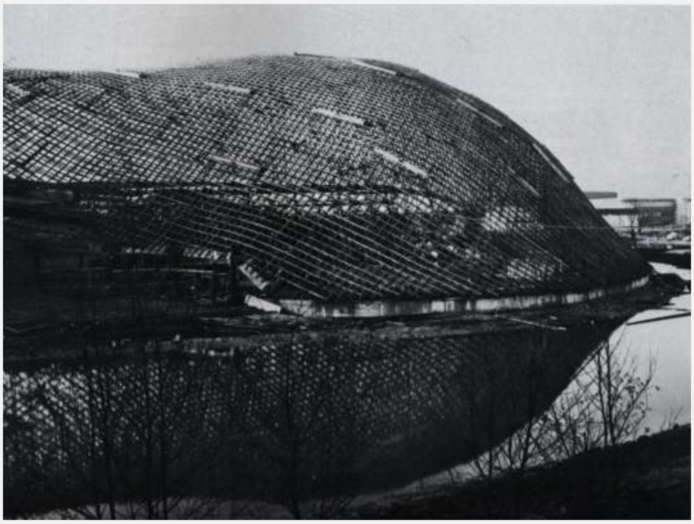


1.1c

1.1d

1.1 Forming process of the gridshell of Mannheim, Germany
1.1a Assembly of the timber grid
1.1b Deformation of the grid
1.1c Final shape of the lattice
1.1d Roofing with a membrane

that takes advantage of the flexibility inherent to slender elements. A regular planar grid made of long continuous linear members is built on the ground (see fig. 1.1a). The elements are pinned together so the grid has no in-plane shear stiffness and can accommodate large-scale deformations during erection. Then, the grid is bent elastically to its final shape (see figs. 1.1b and 1.1c). Finally, the grid is frozen in the desired shape with a third layer of bracing members and the structure becomes a shell. This process is illustrated and detailed in the next chapter (see §2.3).

1.2.2 STRUCTURAL TYPOLOGY

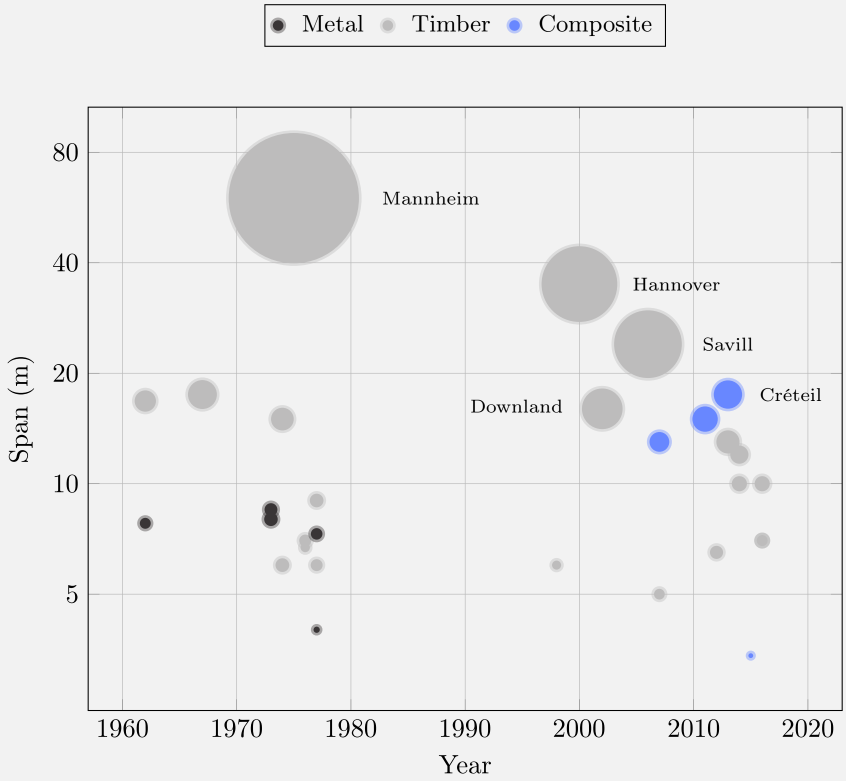
Their mechanical behaviour is very similar to the one of real shells even if the material is discrete and located in a grid more or less open. Moreover, gridshells benefit from the same advantages as the ones showed by an eggshell : they can cross large span using a low amount of material. Their stiffness is mainly linked to their double-curved shape.

1.2.3 MATERIAL FLEXIBILITY FOR STRUCTURAL RIGIDITY

In this field of application, composite materials like glass fibre reinforced polymer (GFRP) could favourably replace wood, where both resistance and bending ability of the material is sought [7]. The stiffness of the structure does not derive from the intrinsic material rigidity but principally from its geometric curvature. Ideally, the composite profiles are produced by pultrusion, an economic continuous moulded process. The standardisation of the process guaranties very stable material and mechanical properties. It frees designers from the painful problematic of wood joining and wood durability. The characterisation of this material is presented further in the thesis (see §2.5).

1.3 BUILT ELASTIC GRIDSHELLS : A REVIEW

No thorough historic review is available about executed projects of elastic gridshells although some partial reviews have been done time to time on the occasion of scientific works or construction projects. This review aims at filling this gap by giving an overview of the development of the concept from the very beginning to the very last experiments. Only known built projects have been identified and reported here. The only condition for a project to belong to this review is to comply with the definition of what an elastic gridshell is (see §1.2), independently to any other consideration (material, fabrication, size, cladding, ...).



1.2 Known elastic gridshells built by the past

The surface of the bubbles is proportional to the covered area. Colour indicates the material employed for the rods.

The informations collected during this research work are given in table format in appendix (see chapter A). A synthetic presentation of these datas is proposed to the reader in fig. 1.2, where projects are ordered by date, span, covered area and material.
The books edited by the *Institut für leichte Flächentragwerke* are of great interest to understand the beginnings. *IL10 Grid Shells* [19] has a precise inventory of the first experiments from 1962 to 1976, while *IL13 Multihalle Mannheim* [1] focuses on the construction of the Multihalle in Mannheim. *Timber gridshells: architecture, structure and craft* [20] is a significant effort but focuses exclusively on medium to large scale projects in timber. A small but general partial review is also available in [21]. An interesting review is also given by Quinn and Gengnagel [22] as part of their research work on new erection methods. A review of bracing and cladding systems is done in [23]. A review of form-finding methods is done in [24]. Finally, various valuable reviews are available in the thesis of Douthe [25], Bouhaya [26], Tayeb [27], and Lafuente Hernández [28].

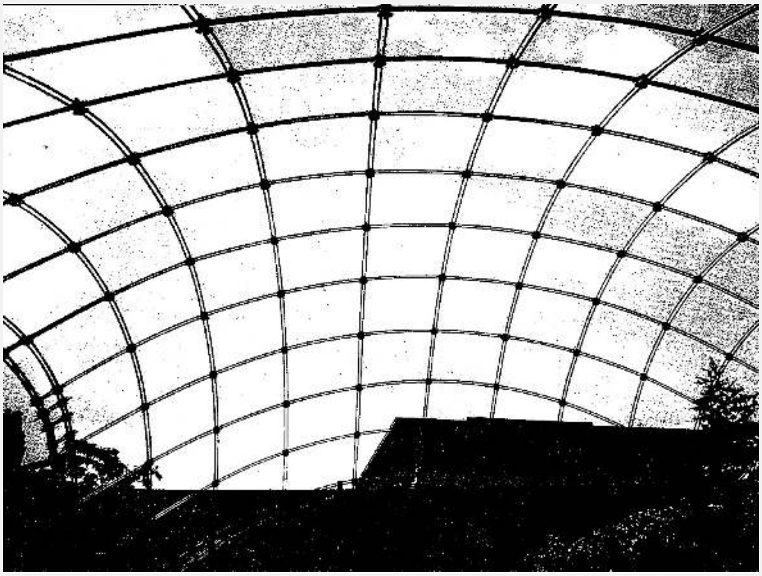
1.3.1 THE BEGINNINGS : FROM THE FIRST PROTOTYPE TO THE GERMAN PAVILION

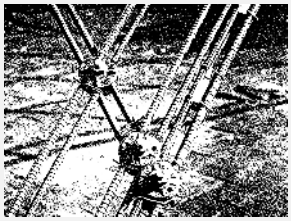
Frei Otto started his studies in architecture in 1947 in Berlin, Germany, and completed his doctorate on tensile structures in 1953. This first work was published and translated later in the 60's. He then began to work in the field of lightweight structures using physical models such as soap films or hanging nets, and photographic measurements.[1,2] These tools were essentials for his exploration of forms and structures as there were no computers at that time.

Steel Gridshell, Berkeley, USA, 1962

Simultaneously, he became interested by the study of lightweight shells and the way they were form-found. One of his very first elastic gridshell was built in 1962 with students at Berkeley, USA [19, p. 270]. It is funny to remark that this first gridshell was not a timber gridshell but a steel gridshell made out of twin steel rods linked in a grid fashion by bolts with clamping plates (see fig. 1.3a). This first experiment demonstrated at small scale the

1. In the 19th and 20th centuries model testing was at the heart of structural innovation [29]. Analog models were employed successfully by well-known architects and engineers to go beyond the limits of existing knowledge (A. Gaudi, H. Isler, F. Candela, F. Otto, …) and are still employed today where numeric models failed to represent accurately some physical phenomenons (for instance in wind analysis for high rise towers and bridges).
2. "Photography is the medium through which the form and content of a model are communicated. It is one of our most important tools in that it provides the basis for documentation and information, supplements our creative potential […]" [19, p. 56]


1.3a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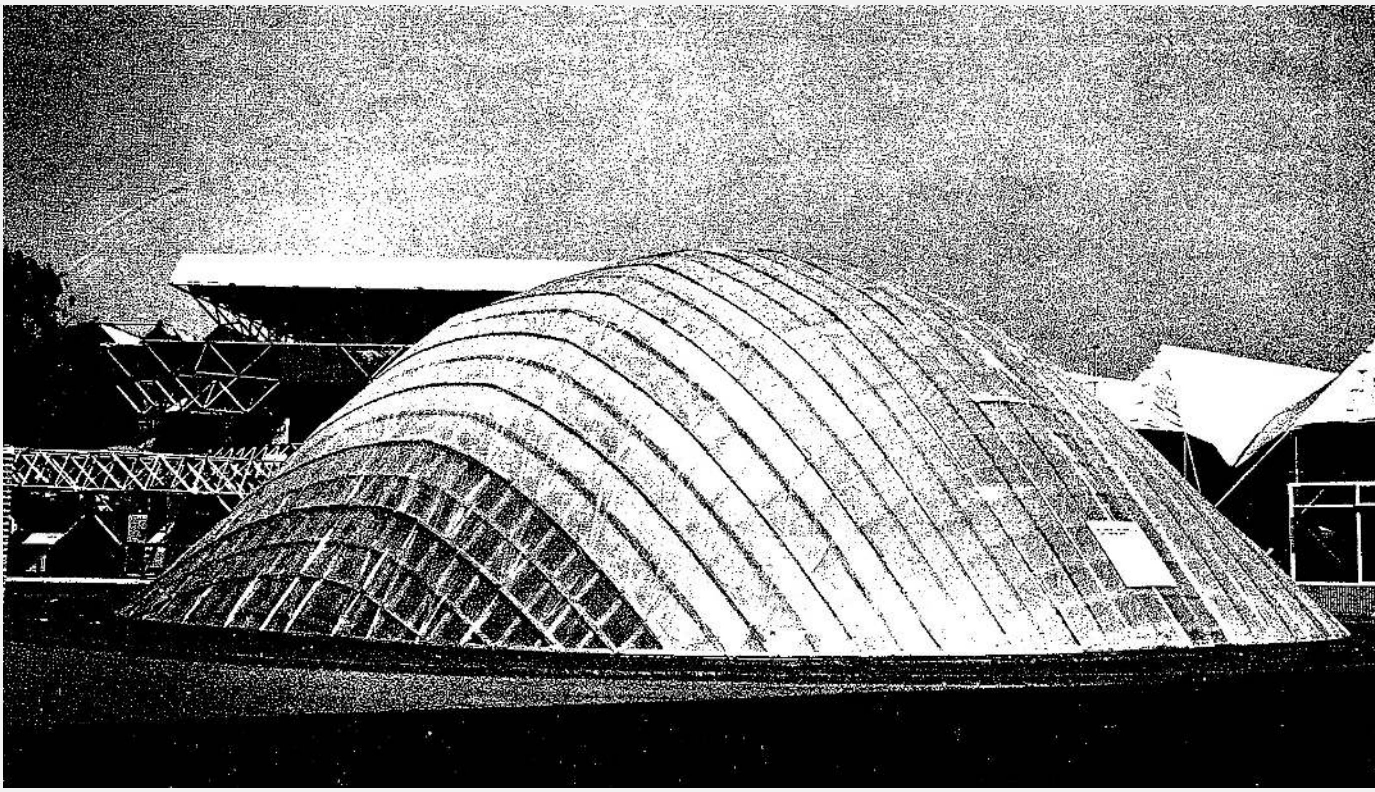
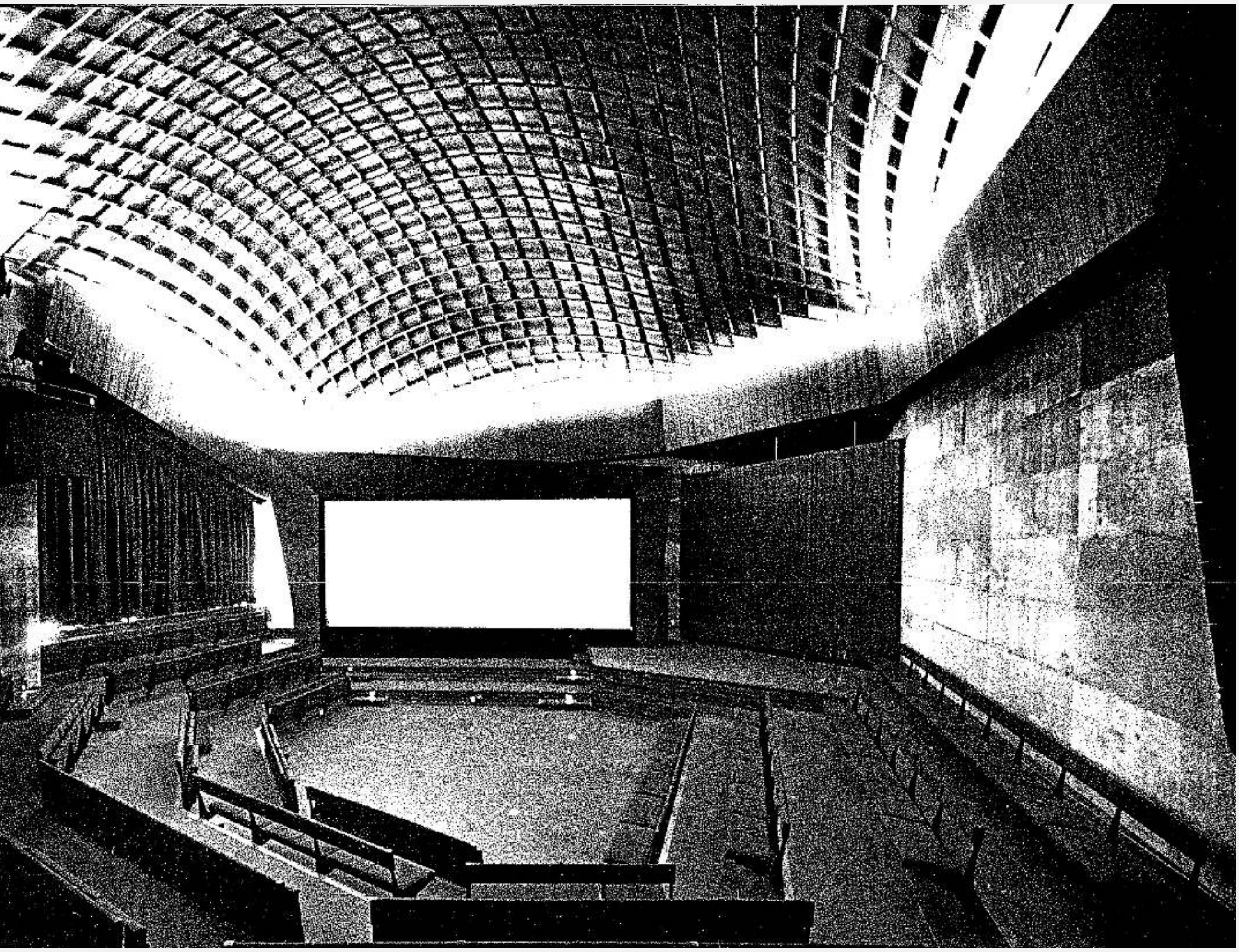

1.4



1.5

1.3 Steel gridshell built in 1962 in Berkeley, USA
1.3a Steel lattice
1.3b Knot detail

1.4 Timber gridshell built in 1962 in Essen, Germany
1.5 Timber gridshell built in 1967 in Montreal, Canada

ability to bend a regular grid with no shear rigidity into a curved shape (see fig. 1.3b). The grid was loosely braced and shell effects were not investigated.

Essen Gridshell, Essen, Germany, 1962

The same year he designed and built a first timber gridshell in Essen, Germany [19, p. 272]. The prototype – a single-layer gridshell spanning 17 m and covering an area of 198 m^2 – was made with 3-plies laminated timber profiles in hemlock pine (see fig. 1.4). The cross-section of the profiles was rectangular (60 mm x 40 mm) and the elements were assembled in a grid fashion with simple steel bolts. Once erected, nothing was specifically done to improve the in-plane shear stiffness of the grid and activate a shell behaviour. Finally, the structure was covered with a transparent plastic foil nailed directly on the grid's profiles.

German Pavilion Auditoria, Montreal, Canada, 1967

Five years later, on the occasion of the *1967 International and Universal Exposition* in Montreal, Canada, Frei Otto was appointed to design the German Pavilion : a large cable net tent prefiguring the realisation of the olympic stadium of Munich, Germany, in 1972.[3,4] The pavilion required two auditoria and these were designed using the principle of elastic gridshell [19, p. 274]. All together, the auditoria covered and area of 365 m^2 and spanned 17.5 m. The construction technique employed in Montreal was quite similar to the one developed in Essen, but this time the grid was fully braced with a layer of nailed plywood boards and offered a proper roofing made out of insulation panels covered with a PVC coated fabric (see fig. 1.5).

The two gridshells built in Montreal mark a significant step in the maturation process of the technique leading to the major realisation of Mannheim in 1976 : a methodology has emerged to progress "from the inverted form to the gridshell" [19, p. 179] ; main construction details have been validated ; various erection methods have been tested ; mid-scale buildings have been built to host public. However, due to the over complexity of these structures, lots of unknowns remained unsolved at this stage and the behaviour of the structures could not be fully predicted.[5]

3. Actually, Frei Otto became the director of the newly founded *Institute for Lightweight Structures* (Institut für Leichte Flächentragwerke or IL) at the University of Stuttgart in 1964. It was the IL that was commissioned by the German government to conduct research in connection with the planning of the German pavilion for the exposition in Montreal.
4. Video of the construction of the German pavilion : https://www.youtube.com/watch?v=Z0mtFMoseUk.
5. "Snow accumulations in the throat of the common edge beam probably caused one of the two grid shells of project Montreal to buckle in a relatively flat region. The diameter of the buckled area was about 3 meters. Neither grid rod was

It is worthwhile to mention that several unexecuted large-scale projects were studied by Frei Otto between 1967 and 1973 at the *IL* or at the *Atelier Warmbronn*.[6] These projects are basically documented in [19, pp. 278 - 288] and reveal that he was training his capacity to master large-scale projects with the technique of elastic gridshells for more conventional building projects (wave pool, swimming hall, multi-purpose hall, auditorium, ...).

1.3.2 MANNHEIM MULTIHALLE : THE COMPLETION OF A DECADE OF RESEARCH

The project of the Multihalle started in 1970, when the decision was made that Mannheim, Germany, would hold the Bundesgartenschau in 1975.[7] The architects of the project, *Carl Mutschler & Partners*, consulted Frei Otto at *Atelier Warmbronn* as he was starting to get known in the filed of innovative lightweight structures. This is how the idea of the gridshell was introduced in the project [30].

A thorough report on the project is available in [1]. A more condensed but still precise description of the engineering problematics related to this project are available in the excellent papers from Happold and Liddell [2] and Liddell [30].

Multihalle, Mannheim, Germany, 1975

Mannheim is an unprecedented realisation because it is more than twenty times larger than the previously built gridshells in Montreal and is meant to last many years and not only for the duration of a short-term exhibition. The timber lattice, still existing in 2017, covers an area of 7400 m^2 (see fig. 1.6a). It is composed of two interconnected domes, one for the multi-purpose hall (span : 60 m | height : 20 m) and one for the restaurant (span : 50 m | height : 18 m).

Although the constructive system deployed in Mannheim clearly inherited from the previous developments, the challenge was such that it had to be revisited. In particular the main additions were the introduction of the double-layer system and the proper bracing of the grid. A major advance was also the use of the very first numeric models to study the structure.

The double-layer system was introduced to tackle two issues : the grid needed some flexibility to be bent into the desired shape, but once erected it should provide sufficient

broken, i.e. the buckling progressed elastically. It might have been possible to press the buckled area back into shape." [19, p. 219]

6. Atelier Warmbronn is the architectural studio founded by Frei Otto in 1969.

7. The Bundesgartenschau is a national horticultural exhibition that takes place every two years in Germany.

1.6a

bending stiffness to resist disturbing loads and avoid a buckling collapse.[8] Once erected, the two grids, one sliding on top of the other one, were connected together to form a single grid with much higher ladder profiles (from 50 mm to 150 mm), increasing their bending stiffness by a factor of about 26 (see fig. 1.6b).

Because the in-plane stiffness of the grid also plays a major role in the resistance to buckling, this question was considered with care. The bracing of the grid was first achieved by preventing the nodes to turn once the grid was erected. This was done by creating some friction in the nodes when tightening the bolts linking the laths, after the grid was erected. Then, additional bracing cables were put in the grid.

8. Theoretically, self-weight loads would produce only compression in the members because the (funicular) form of the grid resulted from the inversion of a hanging chain model in pure tension.


1.6b

1.6 Timber gridshell built in 1975 in Mannheim, Germany
1.6a Sky view
1.6b Interior view

Finally, the project of Mannheim was a key project in the development of modern lightweight structures. Great engineers were born in touch with Frei Otto, following his footsteps or collaborating with him. This heritage has irrigated for several decades the engineering of lightweight structures in Europe and gave birth, directly or indirectly, to several studios among which we can cite *Buro Happold* and *Schlaich Bergermann & Partner*.

1.3.3 THE DRY PERIOD : 25 YEARS FROM MANNHEIM TO HANNOVER

Although the experience of Mannheim proved the feasibility and the potential of gridshell structures for large-scale projects, it also revealed that these projects were subject to an incredible complexity in terms of structural design, geometry, modelling, testing, team work, construction methods ... At that time, very few people could pretend to master all the knowledge and techniques required to design and built timber gridshells and developed in the bosom of the *Institute for Lightweight Structures* in Stuttgart.

This project was obviously well ahead of its time and the engineering cost to design such structures was probably prohibitive considering the tools available at that time. This certainly explains why no elastic gridshells were built during the 25 following years, despite the optimism of the pioneers of the Multihalle.[9]

Note that around 1975 small workshop and experiments lead to the construction of several but small elastic gridshells, as reported in [19]. A non-exhaustive but quite extensive list of known executed gridshell projects is presented in fig. 1.2. The dry period is clearly visible.

1.3.4 THE SIGNS OF A RENEWAL : DORSET AND DONCASTER

It is only 20 years later that gridshells started to reappear, in the late 90's mainly in the United Kindom, and for projects that had interest in environmental problematics.

Westminster Lodge, Dorset, England, 1995

In 1995, a small student residence named *Westminster Lodge* was built in Dorset, England. This dwelling was part of a larger project – Hooke Park – aiming at investigating how the local forest resources, in particular immature roundwood thinnings, could be better

9. "For many years after its completion, Happold promoted the benefit of the timber gridshell as a construction technique and stated that he could not understand why it had not been adopted more widely. He perceived the benefits to be in the efficiency of the construction method to enable doubly curved (shell) structures to be constructed quickly and cost effectively." [31].

1.7a

1.7b

1.8a

1.8b

1.7 Roundwood gridshell built in 1995 in Dorset, England
1.7a Interior view
1.7b Exterior view

1.8 Timber gridshells built in 1998 in Doncaster, England
1.8a Interior view
1.8b Exterior view

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APPLICATION TO THE DESIGN OF ELASTIC GRIDSHELLS

An *elastic gridshell* is a freeform structure, generally doubly curved, but formed out through the reversible deformation of a regular and initially flat structural grid. Building curved shapes that may seems to offer the best of both worlds : shell structures are amongst the most performant mechanically speaking while planar and orthogonal constructions are much more efficient and economic to produce than curved ones. This ability to "form a form" efficiently is of peculiar importance in the current context where morphology is a predominant component of modern architecture, and envelopes appear to be the neuralgic point for building performances.

The concept was invented by Frei Otto, a German architect and structural engineer who devoted many years of research to gridshells. In 1975 he designed the Multihalle of Mannheim, a 7500 m^2 wooden shell which demonstrated the feasibility of this technology and made it famous to a wide audience. However, despite their potential, very few projects of this kind were built after this major realization. And for good reason, the resources committed at that time cannot guarantee the replicability of this experiment for more standard projects, especially on the economic level. Moreover, the technics and methods developed by Otto's team in the 1960s have mostly fall into disuse or are based on disciplines that have considerably evolved. New materials, such as composite materials, have recently emerged. They go beyond the limitations of conventional materials such as timber and offer at all levels much better technical performances for this kind of application. Finally, it should be noted that the regulatory framework has also deeply changed, bringing a certain rigidity to the penetration of innovations in the building industry. Therefore, the design of gridshells arises in new terms for current architects and engineers and comes up against the inadequacy of existing tools and methods.

In a first part, we deliver a thorough review of this topic and we present in detail one of our main achievements, the ephemeral cathedral of Créteil, built in 2013 and still in service. In a second part, we develop an original discrete beam element with a minimal number of degrees of freedom adapted to the modeling of bending and torsion inside gridshell members with anisotropic cross-section. Enriched with a ghost node, it allows to model more accurately physical phenomena that occur at connections or at supports. Its numerical implementation is presented and validated through several test cases. Although this element has been developed specifically for the study of elastic gridshells, it can advantageously be used in any type of problem where the need for an interactive computation with elastic rods taking into account flexion-torsion couplings is required.

In this thesis, which marks an important step in a personal research adventure initiated in 2010, we try to embrace the issue of the design of elastic gridshells in all its complexity, addressing both theoretical, technical and constructive aspects.

PhD