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# Reciprocal Shades: A computational workflow for knowledge-based design and fabrication of multi-performance reciprocal systems

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## Abstract

This research paper presents a novel computational workflow for design, analysis and fabrication of multi-performance reciprocal systems with planar elements. The proposed method integrates dynamic form-finding process, structural analysis and digital fabrication of non-conventional reciprocal systems. Using a multi-step feedback system, simulation feedback and fabrication requirements are integrated in the design process to ensure the performance requirements and constructability of design solutions. As an integrative design process this workflow extends the state of the art in the following ways. A novel mesh-based pattern generation formulation is implemented for design and patterning of freeform reciprocal systems. A dynamic form finding process optimizes the reciprocal patterns to accommodate for geometric constraints of planar members. A simplified formulation for structural analysis and design is used for interactive analysis of the structure. Through a series of physical testing and computational simulations a minimal connection detailing is designed to reduce connection complexity and also facilitate assembly process. The proposed workflow is tested through a full-scale prototype fabrication in form of a public installation to be located in Matthaei Botanical Gardens in Ann Arbor, Michigan. This research is a basis for design and fabrication of innovative lightweight reciprocal systems made of sustainable materials for applications in building façade systems.

**Keywords:** reciprocal structures, computational design, knowledge-based design, performance-oriented design, digital fabrication, wood structures, modular systems, lightweight structures

## 1. Introduction

The principle of reciprocity is based on 3D assembly of loadbearing members that mutually support each other along their span and create a self-supporting spatial configuration without any structural hierarchy which can span multiple times the length of members.

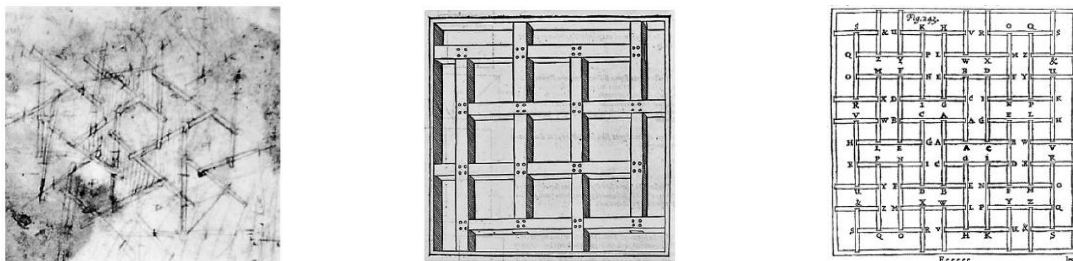


Figure 1: Left) Sketches by Leonardo da Vinci, middle) aligned axis floor system Sebastiano Serlio, right) floor system by John Wallis

The history of these systems probably goes back to one thousand years ago and the tradition of wooden arch bridge fabrication in China or spiral roof structures in Japanese houses (Pugnale and Sassone [1]).

This construction method was adopted in the west and developed to flat configurations like the reciprocal patterns in drawings of Leonard da Vinci or aligned axis grillage floor system by Sebastiano Serlio or notched elements designed by John Wallis (figure. 1).

The grillage mechanism in the reciprocal systems comes from mutually supporting members forming closed circuits (called reciprocal modules) comprised of more than three members. Each member is also part of the adjacent circuit on its other end.

The reciprocal systems encompass huge variety of structural forms depending on number of modules, number of members in reciprocal modules, regularity or irregularity of reciprocal modules, planarity or non-planarity, and single or varying curvature of the global geometry. However, the governing concept and behavior of these systems are the same. In this research we show how a unified computational model is developed to geometrically model comprehensive variations of reciprocal forms.

## 2. Modeling

### 2.1. Morphology of reciprocal systems

In order to explain the modelling methodology, we first define the design parameters (figure. 2). The modelling procedure starts with pattern generation and form finding of 1D members (rods). The design parameters in this stage are the number of reciprocal modules and engagement length of members in modules. Using these two design parameters the reciprocal structure with 1D elements is created and is used as the center-line for generation of 2D members. Then, a rotational parameter is introduced to rotate the planar members around their longitudinal axis based on their corresponding cell. These rotations control the depth of the reciprocal modules which is important for integration of environmental performances such as shading and lighting (Oliyan Torghabehi et al. [2]).

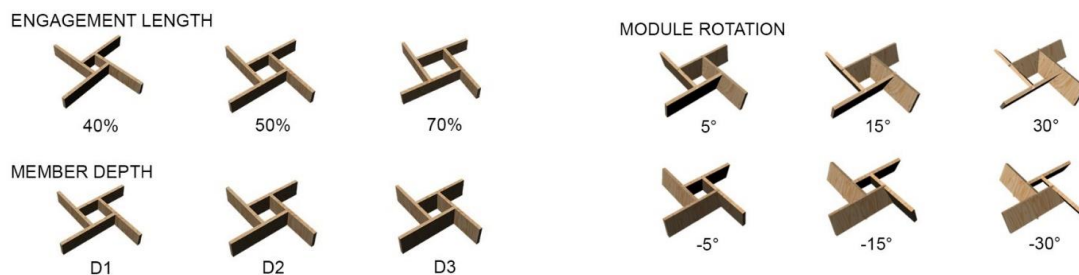


Figure 2: Three main design parameters for reciprocal structures with planar elements.

Different modeling approaches have been developed for geometric modelling of reciprocal systems which can be categorized in two groups, first group generates a 2D pattern of reciprocal system and projects it on the 3D geometry using conformal mapping (Song et al. [3]), Although this method is convenient to implement however it cannot be applied to closed multi-curvature systems. The second method uses discretized geometry of a surface in form polygonal meshes and generates the reciprocal pattern by translating or rotating the mesh edges (Douthe and Baverel [4]). This methodology is more applicable to complex geometric forms however does not provide information on reciprocal modules and their corresponding members, this information can significantly be helpful when dealing with integration of fabrication information into the design process.

The proposed methodology for modelling of reciprocal systems is an extended version of cell-based formulation first developed by Anastas et al. [5], which uses mesh-based rationalization of the surface geometry. However, in the method proposed here, a new formulation is implemented for reciprocal pattern generation using mesh information which provides better control on the reciprocal modules and creates a data structure based on mesh data which is needed for 2D and 3D member generation and also better implementation of fabrication parameters (figure. 3).

This new methodology is applicable on geometries with different levels of complexity and variations in curvature. Also, through using half-edge data the formulation is applicable to non-ordered mesh geometries as well.

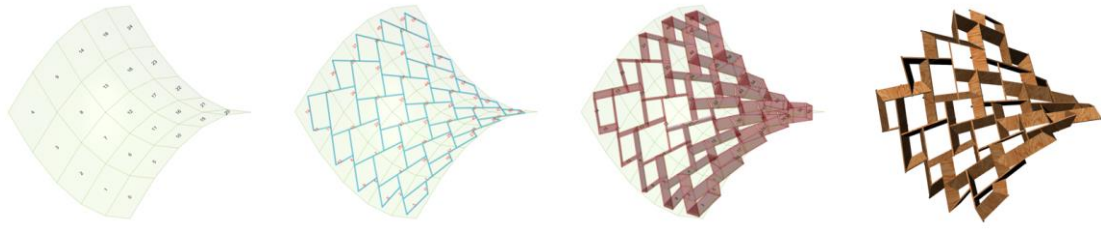


Figure 3: Mesh-based modelling procedure for reciprocal structures with planar elements.

## 2.2 Optimizing the eccentricities of reciprocal members

Using the patterning formulation, the 3D geometry of the reciprocal system is generated, however, due to the existing curvature in 3D geometries the reciprocal members would not be tangent to each other. Therefore, the members need to move in a way that they all connect to each other. Towards this goal different methods have been developed. Song et al. [3] proposed an optimization method which solves the least square formulation of the distances between the rods. However, a more interactive method is proposed by Douthe and Baverel [4] and later developed by Senatore and Piker [6] using Dynamic Relaxation method. In this method we consider the members as rigid bodies and we define the distance between the members as zero length springs and find the minimum strain energy configuration through iterative solution of dynamic stability using Dynamic Relaxation. Since we are using the central line of the members for generation of the 2D and 3D members we would like to have the reciprocal members to touch each other in a 3D space. Towards this goal we use physical solver developed by Senatore and Piker [6]. Margin of error in this method depends on the curvature of the geometry as well as fineness of the rationalization which can be taken in to the consideration when implementing the fabrication parameters (figure. 4).

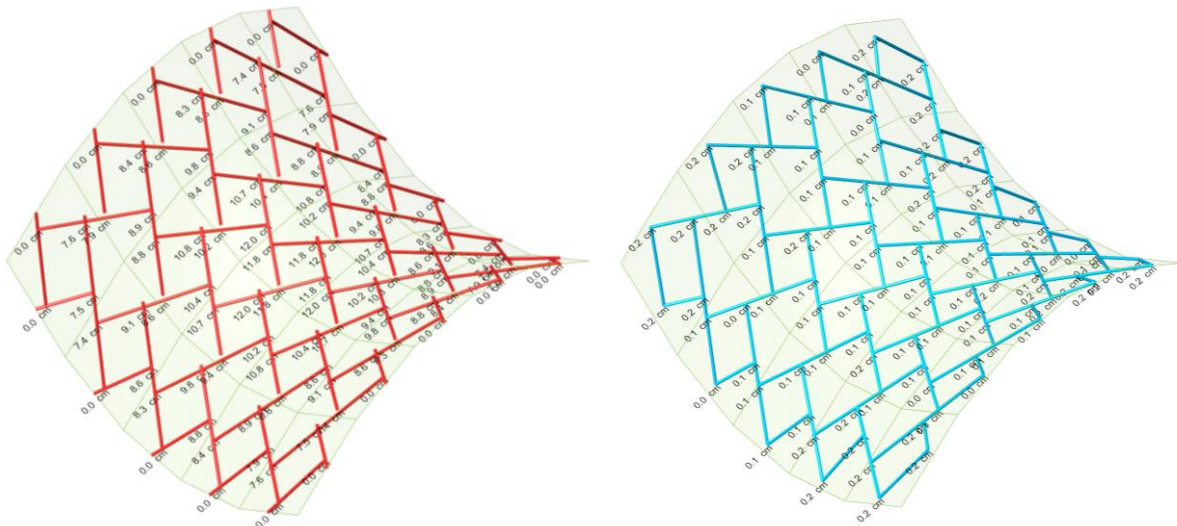


Figure 4: Computational process for minimization of eccentricities of reciprocal members using dynamic relaxation. Left) Initial reciprocal member configuration, right) optimized reciprocal member configuration.

Once the final position of the members was determined the members central axis is generated and trimmed by the intersecting members. These lines are used to generate the 2D members based on the member thickness and members depth parameters. Also using the rotation parameters these 2D elements are rotated to control the perforations in the reciprocal modules. The direction of the rotation for each member is defined based on the corresponding reciprocal module that the member belongs to on each end. This data structure also accommodates local variation of reciprocal modules using attractors or agent-based modelling to generate a more responsive geometric variation based on the desired performance or visual aesthetics (figure. 5).



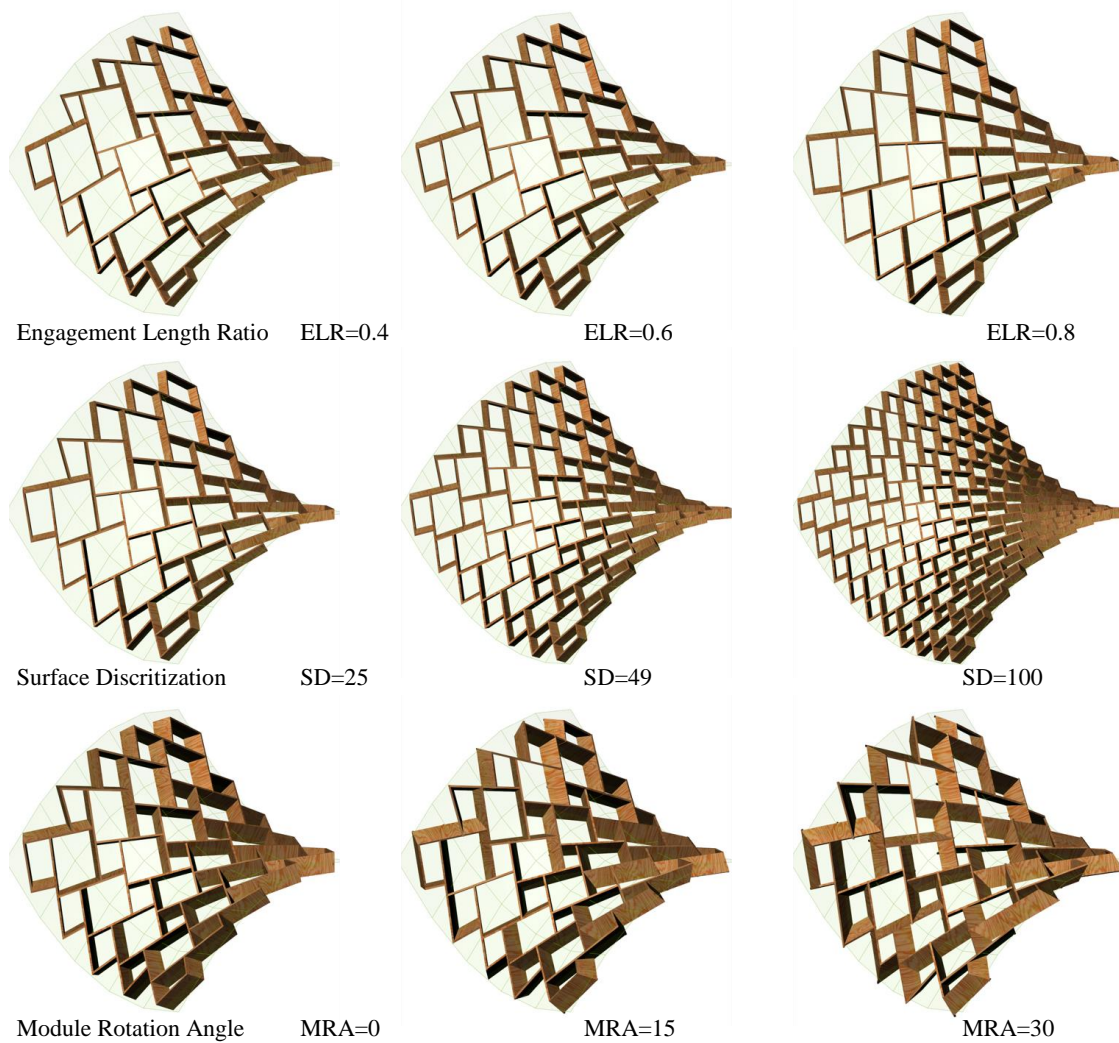


Figure 5: Parametric variation of reciprocal structure based on design parameters.

### 2.3. Analysis of reciprocal systems

In this paper we use simplified Finite Element method for analysis of the structure. This method divides the reciprocal members at the intersection points with other members and creates structural beam elements for each piece. These beams have moment resisting connection to the adjacent piece unless they are at the two ends of the reciprocal member where the member pieces are connected to other members in the reciprocal module which due to nature of that connection detailing is considered a pinned connection. We use Karamba 3D (a structural analysis and design for Grasshopper) for structural analysis of these members. This simplified modelling produces accurate analysis results and also facilitates fast interactive structural feedback (figure. 6).

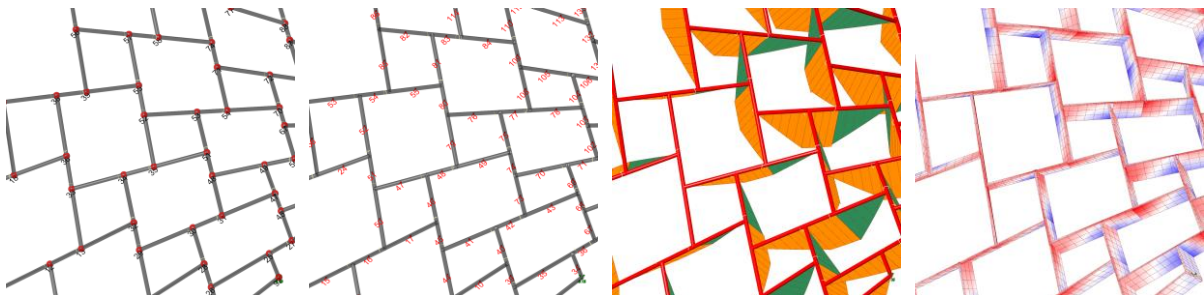


Figure 6: Analytical model and analysis results. from the left) structural nodes, member discretization, moment distribution, member utilization factor (false color).

## 2.4. Fabrication detailing and parameters

The most important fabrication issue is a consistent connection detailing. There are some considerations regarding designing the connection detailing that should be taken into account for a practical fabrication and construction of self-supporting irregular reciprocal systems with 2D members (Baverel and Pugnale [8]). The connection detailing should provide enough structural integrity to connect members in the reciprocal modules, also the connection should be fabricated in a way that simplifies the assembly of the members when small fabrication tolerances are desired. Also, the detailing should have specific mechanism to guide the connectivity of rotated member with varying connection angles. Towards this goal and through series of fabrication tests a revised notched connection detailing is developed with adaptable notch depth which responds to internal forces of the members as well as guides the connection of the rotated members, Also two screws are used with an angle to the plane of each member at the two ends, connecting to the intersecting member and creating a rigid geometric assembly (figure. 7).

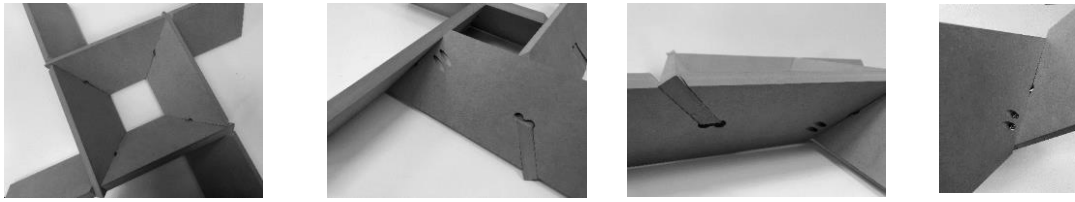


Figure 7: Fabrication detailing for planar member connections.

The behavior of these connections is tested through series of structural tests as well as detailed Finite Element modelling of the reciprocal module. These tests show how the screw connection behaves specially in response to the torsional moments induced by the asymmetry of the reciprocal configurations. Also, this information is critical in understanding the boundary conditions of member connections in large scale simulation using beam elements (figure. 8).

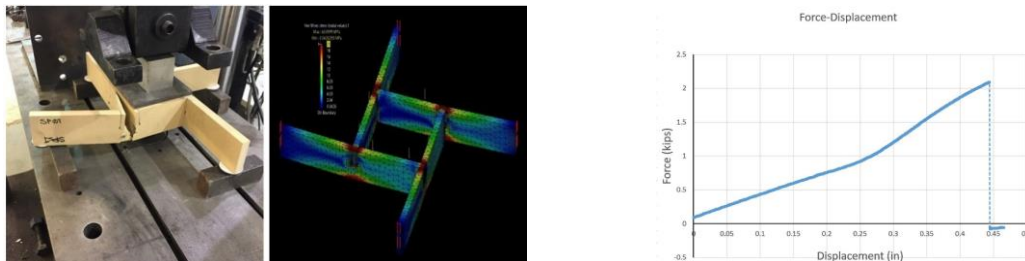


Figure 8: Connection testing. Left) physical structural testing, detailed FEM analysis, force displacement diagram.

## 3. Methodology

We propose a systematic approach for implementation of the modelling, simulation and fabrication knowledge in the design and fabrication of reciprocal systems with planar elements. We use Building Object Behavior notation for development of our knowledge-based method (Cavieres and Al-Haddad [9]). Towards this goal, first, design parameters are defined, then generative rules and feedback functions are implemented base on the proposed modeling and simulation formulations (figure. 9).

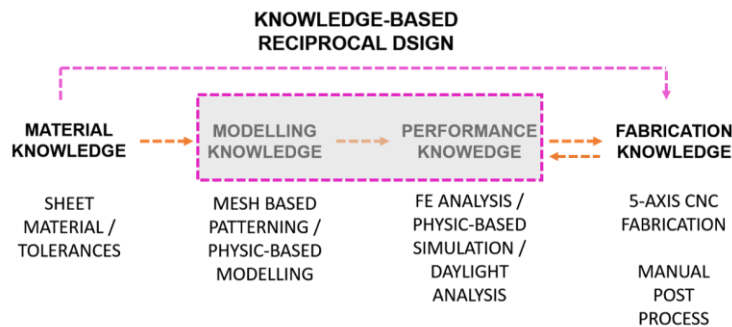


Figure 9: Data flow and the design knowledge.

A consistent work flow is developed to control the data flow and feedback integration in the process. This workflow creates an interactive process for design, analysis and fabrication of reciprocal systems which accommodates complete control on global and local geometric variations while providing instantaneous feedback on structural performance and fabricability of the system (figure. 10).

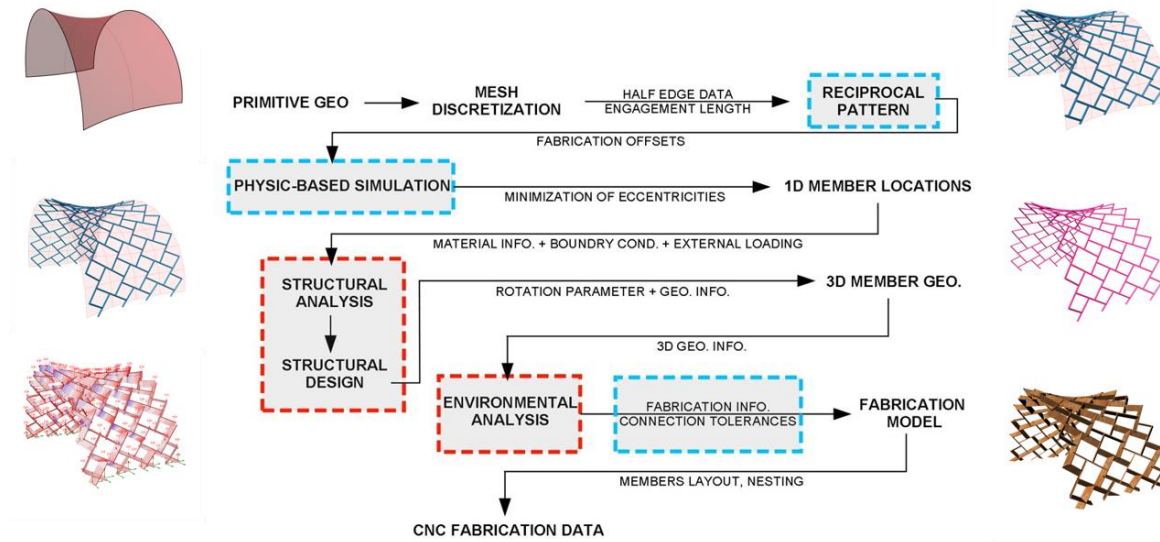


Figure 10: Knowledge-based design process.

Using the proposed methodology, we designed and build a full-scale reciprocal prototype to test the application of the proposed process for large scale construction.

## 4. Design and fabrication of the case study using the proposed method

### 4.1. Arch design and pattern generation and form-finding

The global geometry of the arch is designed based on the site-specific parameters such as dimensions and topography of the site and also the project budget. Quadrilateral reciprocal patterning is used to design the reciprocal system for this project. The quadrilateral patterning provides better control on the perforations shape and distributions which is important in improvement of environmental performances such as shading and lighting. Once the pattern is generated the eccentricities of the members are minimized using Dynamic Relaxation method.

### 4.2. material

For this project 1.9 cm ( 0.75 in) Marine Grade Plywood is chosen for construction of the arch. Plywood is composed of multiple sheets of wood rotated based on the adjacent sheet and glued together. The main benefit of rotating the sheets in plywood is that unlike natural wood plywood panels show consistent strength in all directions. Plywood was chosen for this project due to availability and ubiquity in sheet form, ease of fabrication process using CNC machines, and dimensional consistency which allows small fabrication tolerances. The marine grade plywood has waterproof glue between the sheets which makes the plywood more durable for external use and is more appropriate for this project. Material properties for plywood is provided in table. 1.

Table 1: Material properties of plywood for structural analysis

Property	Specific Gravity (kN/m <sup>3</sup> )	Elastic Modulus (kN/cm <sup>2</sup> )	Shear Modulus (kN/cm <sup>2</sup> )	Tensile Strength (kN/cm <sup>2</sup> )	Compressive Strength (kN/cm <sup>2</sup> )
value	5.99	960	360	1.2	1.68



#### 4.3. analysis

The analytical structural model of the arch is developed based on the proposed simplified method and the material properties are assigned. Supports of the arch is fixed and the external point loads are assigned to the nodes of the reciprocal structure. The structure of the arch is analyzed and designed based on the internal forces which determines the member depth to safely bear the dead weight of the arch plus 1.2 kN/m<sup>2</sup> (25 PSF) distributed external loading which is applied on the arch joints (figure. 11).

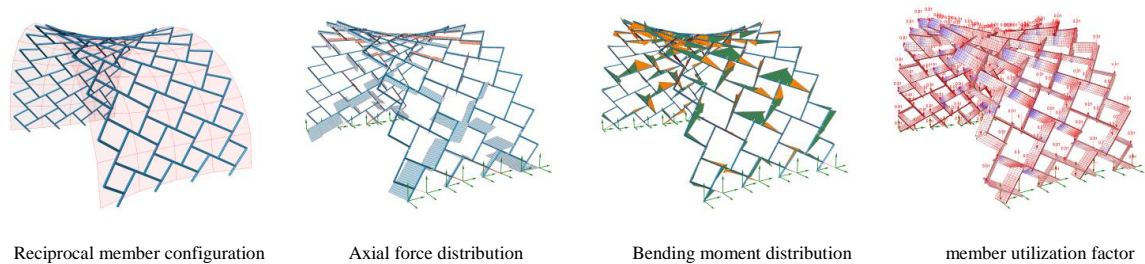


Figure 11: Arch analysis results.

#### 4.4. fabrication process, assembly and results

The proposed fabrication process is enhanced through continuous feedbacks from analysis and form finding process. The proposed workflow provides useful feedback on quantitative metrics such as structural performance, perforations size for better shading and economy of the design based on material use and machining time. The final design is selected based on acceptable performance and aesthetics of the design in relation to the installation site.

The workflow creates fabrication detailing for 5-Axis CNC milling and also labeling information for postprocess and assembly. 112 reciprocal members are cut from 7 sheets of 1.9 cm (0.75 in) Marine Grade Plywood. Screw holes were created manually on each member. And the members are assembled at Taubman College of Architecture before the dome is set up on the designated site in the Botanical Gardens (figure. 12 and 13).



Figure 12: Assembly process.

#### 6. Conclusions

Reciprocal structures historically emerged in different parts of the world at different times mainly to respond to a need which was covering an area bigger than the size of the available timbers. The unique characteristics of reciprocal systems, such as self-equilibrium, modular assembly, inherent three dimensionality and potentials for generative growth, qualify these systems as sources of ideation for innovative design of multi-performance architectural systems.



In this paper we proposed a computational framework for design and fabrication of reciprocal systems with 2D members. Towards this goal a knowledge-based design framework is designed and used for fabrication of a full-scale reciprocal arch with rotated planar members. This research shows how through a knowledge-based process these systems can be designed while responding to material, performance and fabrication needs. The goal of this research is to expand the applicability of reciprocal systems as multi-performance and sustainable architectural system.



Number_Of_Members	112
Minimum_Member_Length (cm)	33.5
Maximum_Member_Length (cm)	106.4
Peak_Height (m)	2.1
Arch_Weight (kg)	136
Displacement (cm)	9.4
Maximum_Utilization	0.356
Engagement_Ratio	0.7
Rotation_Angle (Degree)	25
Member_Depth (cm)	15.2
Member_Thickness (cm)	1.9
Number_Of_4'X8'_Sheets	6.9
Maximum_Formfinding_Error(mm)	2.6

Figure 13: Left) Rendering of the installation at Matthaei Botanical Gardens. Right) Design output information.

## Acknowledgements

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