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Digital modelling of deployable structures based on curved-line folding

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Abstract

Curved-line folding is the act of folding a flat sheet of material along a curved crease pattern in order to create a 3D shape, using the combination of folding (plastic deformation) and bending (elastic deformation). Most applications of curved-line folding only make use of the end state of the folding process: a static solution obtained through folding along a curved crease pattern. However, the elastic deformations that occur when a flat sheet is forced into a curved shape can produce an interesting transformation process. When one surface area is bent, the forces and moments are transmitted through the curved creases to the adjacent surface areas, which results in a folding motion. As a result, this kind of transformation process could be used for the development of a new type of deployable structure, finding its application in the context of kinetic shading systems. The aim of this paper is to give an overview of how this transformation process can be modelled and analysed in a digital environment. Existing methods as well as some new approaches are discussed and evaluated. A distinction is made between pure geometrical modelling methods and simulations with finite elements software. It can be concluded that the existing tools for geometrical modelling of the folding process of deployable structures based on curved-line folding are sufficient to quickly check the deployment of different curved-line folding patterns and can find their application in the early design stage. However, for a more profound analysis, which takes into account material properties and forces, a calculation with finite elements is required.

Keywords: Curved-line folding, curved-crease folding, deployable structures, pliable structures, digital modelling.

1 Introduction

When a very thin material (like paper) is folded along a curved crease, a 3D shape is obtained by folding (plastic deformation) as well as bending (elastic deformation) of the sheet. This principle is called curved-line folding and has been discovered by students from the Bauhaus in the late 1920's, as explained in Demaine *et al.* [1]. Until now, most applications of curved-line folding in architecture only make use of the end state of the folding process. Starting from a flat sheet of material three-dimensional shapes with a geometrical stiffness are obtained, finding applications in sculptures, façade components, furniture etc. Accordingly, the plastic deformation present at the fold lines is permanent and the artefact cannot return to its initial state. However, the authors of this paper believe that curved-line folding can also be used for the design of deployable structures, by use of the elastic deformations that occur when a flat sheet is forced into a curved shape. As one surface area is bent, the forces and moments are transmitted through the curved creases to the adjacent surface areas, which then results in a folding motion. Figure 1 shows how the elastic deformation of the paper model's central area, generated by pushing the ends towards the centre, forces the adjacent areas to fold inwards. In Schleicher *et al.* [2] this phenomenon is referred to as bending-active kinetics and shows great potential for application in the design of adaptive façade shading systems. This paper gives an overview of existing methods and tools to simulate the folding process of curved-line folding patterns in a digital environment. Besides tools that allow digital modelling based on the geometry alone, the authors also present a method using FEM simulation software. This method allows the designer to consider real material properties, actuation forces and boundary conditions when designing a deployable structure based on curved-line folding.

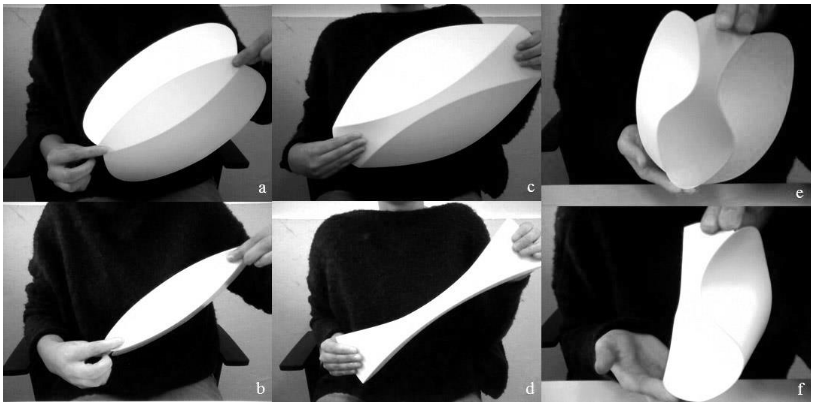

Figure 1: The folding process of various crease patterns is demonstrated, with the open state shown in the upper row and the folded state in the lower row. The folding motion is controlled by the bending of the central area of the models (Vergauwen *et al.* [3]).

2 The geometric nature of curved-line folding

In order to use the principle of curved-line folding for the design of deployable structures it is necessary to first understand the geometric nature of curved-line folding. As explained previously, curved-line folding is the act of folding paper along a curved crease in order to create a 3D shape. Hence, the properties and behaviour of paper lie at the basis of understanding curved-line folding from a geometrical point of view. Paper is a material that can sustain bending without stretching or tearing, just like for example plastic, card-board or metal sheets. Accordingly, sheet materials behave like developable surfaces. Based on the definitions found in Topogonov [4] and Pottmann [5] developable surfaces are characterized by the following properties:

- They can be mapped isometrically into the plane, which means that the surfaces can always be unfolded into the plane while preserving line lengths and angles.
- They are ruled surfaces, with rulings of which the tangent plane touches the surface along the entire line (i.e. torsal rulings).
- They have zero Gaussian curvature (and are therefore also called single curved surfaces).
- Besides the plane, three basic types of developable surfaces exist: the cylinder, the cone and the tangent surface of a space curve.

This means that a random curved-line folded shape is in fact a composition of pieces of these three basic types. In Kilian *et al.* [6] this is explained using the example of Gregory Epps' car design. Figure 2 shows how the 3D shape in the folded state is composed of pieces of planes (yellow), cylinders (green), cones (red) and tangent surfaces (blue). Depending on whether a developable surface consists of pieces of cylinders, cones or tangent surfaces, the rulings are respectively parallel, pass through a common point, or are tangent to a curve (called the curve of regression).

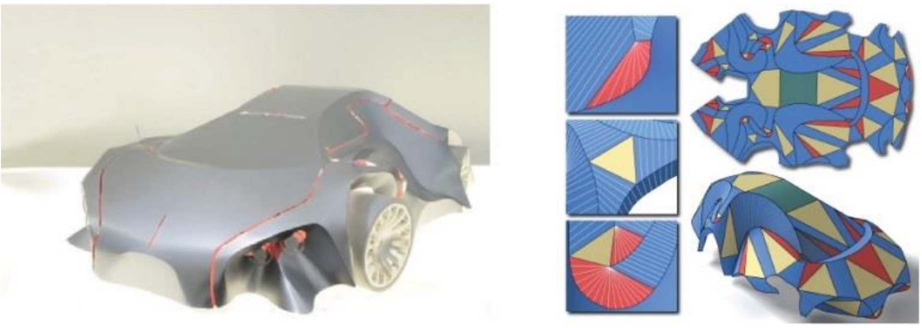

Figure 2: On the left, a conceptual presentation of Gregory Epps'car design (Tachi and Epps [7]) and on the right the same car as a composition of patches of ruled surfaces. Planes are indicated in yellow, cylinders in green, cones in red and tangent surfaces in blue (Kilian *et al.* [6]).

Furthermore, for one unique curved crease pattern, there exists an infinite amount of pairs of surfaces that can be produced by folding around the curved crease. This can be explained by looking at Figure 3. As the paper is folded along the curved line, a basic shape is generated. But by physical interaction with the paper, an infinite amount of twisted variations of this basic shape can be obtained. And for every variation of the basic shape, the position of the rulings is unique. Consequently, the position of the rulings plays an essential role in the geometrical modelling of curved-line folding.

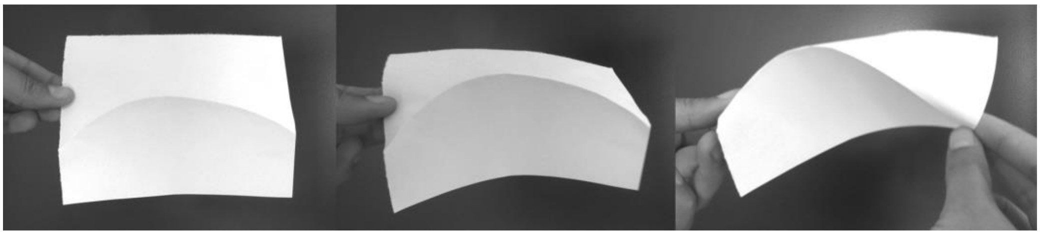

Figure 3: The picture on the left shows the initial state of the curved-crease pattern. The picture in the middle shows the basic shape generated by folding around the curved crease. The picture on the right shows a twisted variation of the basic shape obtained through physical interaction with the paper model.

3 A geometric approach for the digital modelling of curved-line folding

3.1 Differential Geometric Analysis

There are different ways to describe and understand curved-line folding. One way is to study it from a geometrical and mathematical point of view using differential geometry. The first paper describing the behaviour of developable surfaces near creases using this method was written in 1976 by David Huffman [8]. But many others used differential geometry to study the geometric nature of curved-line folding, especially in order to find design and simulation methods [9]–[12]. In [8], Huffman studies the traces of a polyhedral vertex on the Gaussian sphere, to derive several basic relationships among the various (dihedral) angles of the vertex. In addition, he uses the same concept to study the edge of a polyhedron as a discretization of an arbitrarily curved crease. This study led to important observations on the relation between the position of the rulings, the osculating plane and the dihedral angle. In Duncan and Duncan [9] the methods and theorems of differential geometry are used to prove a number of theorems concerning the geometry of folding and to distinguish three special cases of folding: with planar curved creases, with constant folding angle and a combination of both. Based on experimental observations of folding paper, Fuchs and Tabachnikov define a number of theorems on curved-line folding and proof them mathematically in Fuchs and Tabachnikov [12]. In order to thoroughly understand curved-line folding in a geometrical way it is advisable to read all of the works mentioned above, however, the most important theorems are summarized below.

3.1.1 Theorems describing general behaviour around the crease

- In general, neighbouring points on a curved crease will have different osculating planes. Moving a point along a curved crease changes the associated osculating plane's orientation perpendicular to the tangent to the crease [8].
- At every point on the curved crease, the tangent planes of the two surfaces make equal angles with the osculating plane of the curved crease in that point [8] [12].
- The process of curved line folding produces 2 surfaces having equal and opposite surface normal curvatures along the fold line [9].
- Throughout the deformation the geodesic curvature remains equal to the ordinary curvature of the crease in the plane development [9] [12]. As a result, the following equation is obtained:

$$K(t)\cos\alpha(t) = k(t) \tag{1}$$

with K(t) the curvature of the plane curve, α(t) the angle between the osculating plane and the surface and k(t) the curvature of the space curve [12].

3.1.2 Theorems describing special cases

- When the curved crease is planar (the osculating plane is the same in every point), the pair of rulings at any point on the crease makes equal angles with that plane [8].
- When the curved crease remains planar during deformation, then the rulings of each surface are collinear in the plane development [9] [12].
- When the curved crease remains planar during deformation, the surface on one side of the fold is a mirror image in the osculating plane of the continuation of the other surface [9] [12].
- If the folding angle or dihedral angle between the surfaces at the curved crease remains constant along the fold then the rulings make equal angles with the normal to the curved crease in plane development. [9] [12]One of the surfaces is a cone (rulings pass through a single point) and the other surface is a cylinder (rulings are all parallel) [12].
- If all the rulings cross the crease at right angles then the dihedral angle between the surfaces is constant for all points on the crease and the crease lies in a single osculating plane [8]. Both surfaces are cones (and their rulings all pass through a single point) [12].

Although all of these theorems have contributed to a better understanding of the behaviour of rulings along a folded crease, they do not provide a general method to describe the folding process or generate novel forms through simulation. Moreover, only the local behaviour around one single curved crease has been studied, whereas the differential geometric analysis of patterns with multiple creases is still non-existent. However, the insights provided on the effect of the choice of the rulings on the folding behaviour will prove to be very useful when employed in the digital modelling method described in the next paragraph.

3.2 Discrete geometric approach

As explained in paragraph 2, a curved-line folded shape has an infinite amount of twisted variations of its basic shape. For every variation of this basic shape, the position of the rulings is unique. Accordingly, it is only by fixing the position of the rulings and thus discretizing the surface as a planar quadrangle mesh (PQ mesh) that a one DOF mechanism can be obtained. This PQ mesh representation forms the basis for several existing computational methods to represent curved-line folded paper models. In Kergosien *et al.* [10] a mathematical model is presented which allows interactive deformation of a surface remaining isometric to a plane rectangular sheet, like for example the computation of the geometry of a symmetrically crushed can. In [6] Kilian *et al.* presents an optimization based computational framework to compute a digital reconstruction of a physical model and then vary its shape by an as-isometric-as possible deformation. This approach allows for a designer to first build a rough shape using paper or similar materials, which can then be scanned and approximated using an optimization algorithm. Subsequently, the user can modify the digital model using the proposed deformation tools. Although these works are very valuable from a computer engineering and mathematical point of view, they are not really applicable in a design context. Moreover, digital design tools with the emphasis on the folding process and the possibility to quickly check different crease patterns containing multiple creases are needed to allow the (geometrical) design of deployable structures based on curved-line folding. Following, three digital design tools/methods existing today are presented and evaluated.

3.2.1 Freeform Origami © tool

The first practicable simulation method for the design of deployable structures based on curved-line folding was developed by Tomohiro Tachi, by means of his Freeform Origami © tool [13]. The simulation tool provides a smooth interactive animation of folding from a rigid origami crease pattern to a 3D folded shape. As explained in Tachi [14], the origami configuration is represented by folding angles of edges, which are connected with facets to form closed loops. The folding motion is numerically calculated using linear approximation of constraints by single vertices. In order to use the tool for the simulation of the curved-line folding patterns, Tachi proposes to discretize the crease pattern. Accordingly, a curved crease becomes then a polyline and the adjacent surfaces are divided into planar quadrilateral facets using a pattern of rulings. The discretized form obtained is a rigid origami structure with at most one degree of freedom (Tachi and epps [7]). Figure 4 shows an example of a discretized curved-crease pattern folded by means of Tachi's Freeform Origami © tool. As explained previously, the choice of the rulings determines the folding behaviour and the three-dimensional shape. Consequently, the user should have a good understanding of this effect, when discretizing the curved-crease pattern.

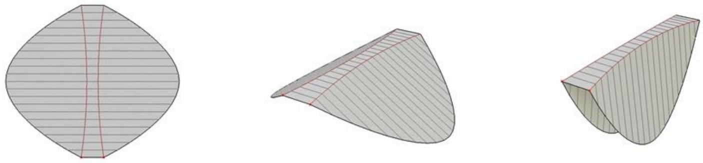

Figure 4: An example of a discretized curved-crease pattern folded using Tachi's Freeform Origami © tool.

3.2.2 Kingkong plugin for Grasshopper

Kingkong [15] is a plugin for Grasshopper developed by Robofold [16] which allows the simulation and parametrical control of an array of curved folded panels and if required, the robotic fabrication by means of the RoboFold production process. Furthermore, the plugin can be used to test façade compositions by means of attractors. This can be very useful for the conceptual design of kinetic shading system based on curved-line folding, like the example shown in Figure 5. Since the plugin automatically creates arbitrarily evenly distributed rulings, the tool can be used without profound knowledge on ruling patterns and their effect on the folding process, which makes the tool very accessible for any user. Although, the Kongmesh component also allows the definition of custom ruling patterns, a more advanced user will quickly bump into some important restrictions. First of all, the script used for the folding simulation isn't accessible, which makes it difficult to influence or understand the folding process. Also, the plugin doesn't seem to work for patterns where the rulings do not continuously pass through all circumscribed surface areas, like for example the pattern of the model shown in Figure 6. It can be concluded that the tool works very well for simple patterns, however due to the protected script it is difficult to make more complex patterns work.

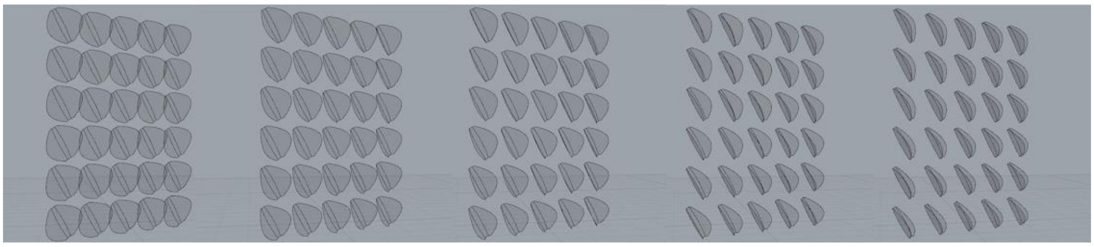

Figure 5: Conceptual design of kinetic shading system based on a curved-line folding pattern. The Kingkong plugin was used to simulate the folding of the model and to test different façade compositions.

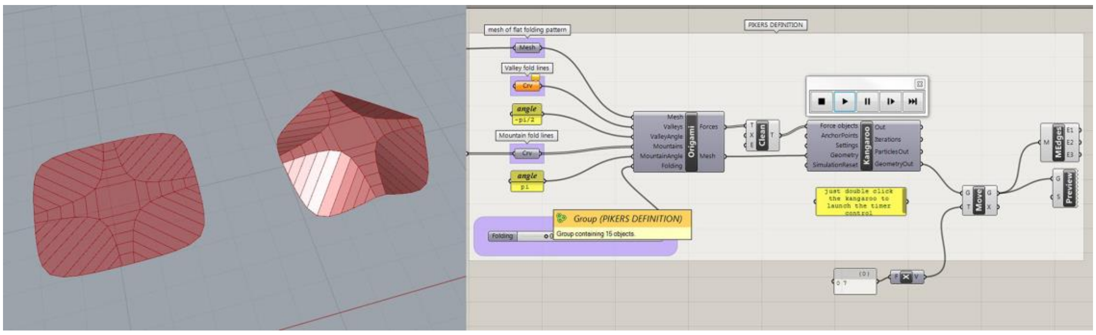

Figure 6: A discretized curved-crease pattern folded by means of Kangaroo and the Origami component.

3.2.3 Kangaroo plugin for Grasshopper

Recently Daniel Piker developed a new component for the Kangaroo [17] plugin for Grasshopper called Origami. The component allows the user to define a mesh and assign the mountain and valley folds. As an output, the component gives a list of forces and a mesh that can then be used as an input for the kangaroo Live Physics engine. The result is a very powerful and easy-to-use tool to simulate the folding of any origami pattern, including discretized curved-crease patterns. A big advantage of this tool is that the Origami component is designed as a Grasshopper cluster with open access. This means that a more advanced user can look into the script, understand how it works and make adaptations if needed. However, a basic understanding of ruling patterns and their effect on the folding behaviour is needed, since the user needs to compose the mesh himself. As shown in Figure 6, a pattern of which the rulings do not continuously pass through all circumscribed surface areas can easily be folded using the Origami component, whereas this was not the case with the KingKong plugin.

3.3 Advantages and drawbacks of the geometric approach

Using a geometric approach for the simulation of the folding process of structures based on curved-line folding allows for a very quick and easy way to understand the folding behaviour and to have an idea about the three-dimensional shape resulting from the crease pattern. In a preliminary design phase this can be a very useful method to do a morphological investigation of the effect of various design parameters (like the curvature of the creases, the type of crease, the composition, etc.). The fact that the tools generate objects in a CAD-environment makes it very easy for designers to implement them in 3D models and renderings. As a result, the geometric simulation of curved-line folding can be very valuable to produce and evaluate design concepts. However, one should keep in mind that this simulation method is an approximation of reality. First of all, a fixed configuration of rulings is chosen, which is not how the rulings are positioned in real life. Furthermore, it is a pure geometrical approach, meaning that no material properties, material thicknesses, forces, stresses or boundary conditions are taken into account. For the design of real deployable structures based on curved-line folding the folding process should thus also be simulated using finite element software. Accordingly, the effect of parameters like the actuation forces, the material properties and thicknesses, the implementation of the creases, stress concentrations, etc. can only be studied through FEM-analysis. In the next paragraph a method for the modelling and analysis of curved-line folding by means of FEM software is explained and evaluated.

4 Digital modelling of deployable structures based on curved-line folding through finite elements simulation

As explained previously, a simulation based on FEM is needed in order to have an idea about the effect of the material properties and thickness, the design of the curved hinge, the boundary conditions and actuation, the stress concentrations and forces acting upon the structure and so on. These are parameters which cannot be verified using the geometric approach. Therefore the authors of this paper describe a method to simulate a number of curved-line folding patterns using a FEM-software called Sofistik [18]. Figure 7 shows the results of some simulations of deployable structures based on curved-line folding. The first row shows the folding of a crease pattern consisting of two arc-curves mirrored in such a way that a concave area is created in the centre. As this central area is bent, by pushing the edges downwards, the flaps fold downwards as well. The second row shows the folding of a crease pattern consisting of two mirrored arc-curves creating a convex central area. In this case, the folding of the flaps is induced by pushing the edges of the central area upwards. The third and fourth row demonstrate the folding of a crease pattern of respectively three and four creases creating a folding motion by pushing the edges of the central area downwards.

4.1 Methodology

Using the principle of paper folding for the design of deployable structures does not mean that the deployment should start from a completely flat state. First of all, the deployable structure requires sufficient geometrical stiffness at all times, also when it is in its most open state. As a result, some curvature is needed in the surfaces in order to withstand external loads (i.e. wind). Furthermore, too much force would be needed to actuate the structure from a completely flat state, leading to uncontrollable deformations and possibly even failure of the structure. Therefore, when the structure is in its most open state, it should already be in a pre-bended phase. Accordingly, the simulation includes two steps, a first step consisting of the pre-bending of the structure in order to guide all the parts in the right direction and a second step to simulate the deployment by means of loads that represent the used actuation system. One way to pre-bend the structure in FEM is to mimic contraction cables. A numerical trick to shorten these contracting cables and thus pull certain areas of the model is to reduce the cable's elastic stiffness under constant pre-stress. This technique of using elastic pre-stressing cables was initially introduced by Lienhard et al. [19] in their research on bending-active structures. As shown in Figure 7 this technique also works for the pre-bending of various curved-crease patterns. The images in the left column show how the cables have been used to pre-bend the central area and the flaps, in the correct direction.

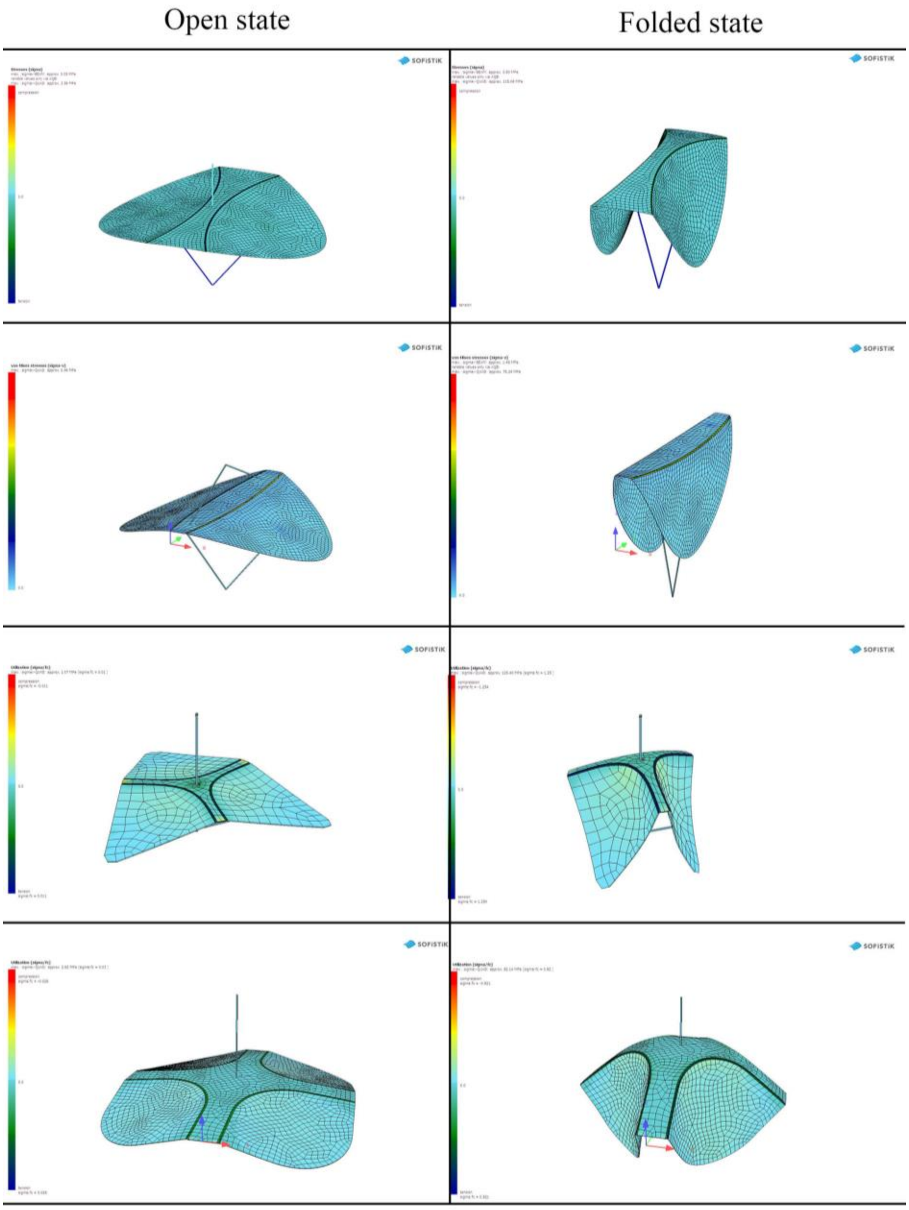

Figure 7: Simulation of four different curved crease patterns. The initial state (after pre-bending) is shown on the left, the compact folded state is shown on the right.

Once the structure has been bent into the chosen initial state, a nonlinear large-displacement algorithm is used to simulate the folding process of the structure. This means that the forces applied on the corner points of the central area of the crease pattern are increased with a pre-defined factor in consecutive steps. As a result, the structure folds until a limit load case is reached. This simulation method allows the user to verify the actuation forces needed for the deployment, which can be used for the design of the actuation system. Furthermore, information on the displacements and stresses in the flaps or in the creases can be obtained. Now that a method for the FE-modelling of curved-line folding structures has been established, a more profound analysis of the kinematic and structural behaviour of this kind of structures can be conducted. Some preliminary tests done by the authors have already shown that the following parameters have an influence on the folding behaviour of the structure: the width of the crease, the thickness of the crease in proportion to the thickness of the sheet, the thickness of the sheet in proportion to the scale of the model, the position and orientation of the actuation forces and the curvature of the crease. In order to get a better understanding of deployable structures based on curved-line folding these parameters should be further investigated.

5 Conclusions

This paper gives an overview of different methods for the digital modelling of deployable structures based on curved-line folding. A distinction is made between computational methods for the simulation of the geometric folding of a crease pattern and an approach for the simulation of curved-line folding as a deployable structure by means of FEM software. It is clear that the geometric approach can be very valuable in the preliminary design phase to test the effect of morphological parameters, façade compositions or obviously whether and how a certain crease pattern folds. Furthermore, all of the tools described in paragraph 3 generate 3D objects in a CAD environment, which gives the user the opportunity to use them in 3D visualisations or renders as part of a design proposal. However, the user should keep in mind that these tools only give an approximation of the real deployment, meaning that no material properties, material thicknesses, forces, stresses or boundary conditions are taken into account. As a result, the method explained by the authors to model curved-line folding structures by means of FEM allows a more profound and realistic analysis. This way, the actuation forces necessary for the deployment can be obtained, as well as information on the displacements and stresses in the flaps or creases, which leads to a better understanding of curved-line folding structures. Even though the digital modelling of deployable structures is a good way to verify a design concept, the real challenge today still remains the translation into a real working deployable system.

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