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UNDULATUS: design and fabrication of a self-interlocking modular shell structure based on curved-line folding

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Abstract

Curved-line folding is the act of folding a flat sheet of material along a curved crease pattern in order to create a three-dimensional shape. It is a creative and innovative way to produce lightweight and geometrically stiff components using only sheet materials. The pavilion presented in this paper integrates this principle in a kit-of-parts system. After being cut out of flat plates, the components get their 3D shape by folding them along a set of predefined curved lines. This deformed geometry, variable in its degree of bending, results in a structurally efficient 'building block' that can be combined into different structural configurations depending on its mode of assembly. The components form a weave pattern by connecting them 'flap to leg' respectively. As such, adjacent components lock each other into their rigid three-dimensional configuration and a modular shell structure is obtained. When composed of identical components, the obtained structure combines the advantages of rapid fabrication and assembly with an extensive reuse of components. However, the pavilion demonstrates that more geometrically and aesthetically challenging compositions, consisting of a series of custom-made components, are also possible and manageable when using digital fabrication techniques. Furthermore, this paper presents and discusses the digital modelling methods used for the design of the pavilion, as well as the lessons learned by real scale fabrication and assembly.

Keywords: curved-line folding, active bending, kit-of-parts, digital fabrication, parametric design, modular shell structure, Grasshopper®

1. Introduction

Curved-line folding is the act of folding a sheet of paper along a pattern of curved creases in order to create a three-dimensional shape. It is a transformation process that combines elastic bending of the sheet through folding of the curved creases. Many artists used this technique to create beautiful and inspiring sculptures out of paper, like the artwork of David Huffman (Demain *et al.* [1]), Ronald Resch, Richard Sweeney (Demaine *et al.* [2]) or Corneel Cannaerts [3]. Although these paper sculptures have a very elegant and humble look, the geometric complexity behind it should not be underestimated and many studies have been carried out to get a better understanding of the geometric nature of curved-line folding (Huffman [4], Duncan and Duncan [5], Fuchs and Tabachnikov [6]). Furthermore, the complexity of the material-dependent transformation process makes the precise digital modelling of objects based on curved-line folding a real challenge. Nevertheless, the authors of this paper saw great potential in this technique and used it as an inspiration for the design of an innovative structural research pavilion. In order to comply with the requirements of compact transportation, easy and quick assembly and re-use of all structural parts, the pavilion combines the principle of curved-line folding with a kit-of-parts system. Starting from a flat and compact state during transportation (figure 1a), each component is bent and folded into its three-dimensional configuration. By mutually connecting the components 'flap to leg' respectively (figure 1b), the components form a weave pattern and are locked into their 3D shape forming a bending-active shell structure. The result is a suspended lightweight cloud-like pavilion with a strong resemblance to the Undulatus Asperatus cloud formation, hence the name of the pavilion (figure 1c). This paper discusses the design and fabrication of the Undulatus pavilion with a specific focus on the digital geometric and parametric modelling and the detailing and assembly of the real scale structure.
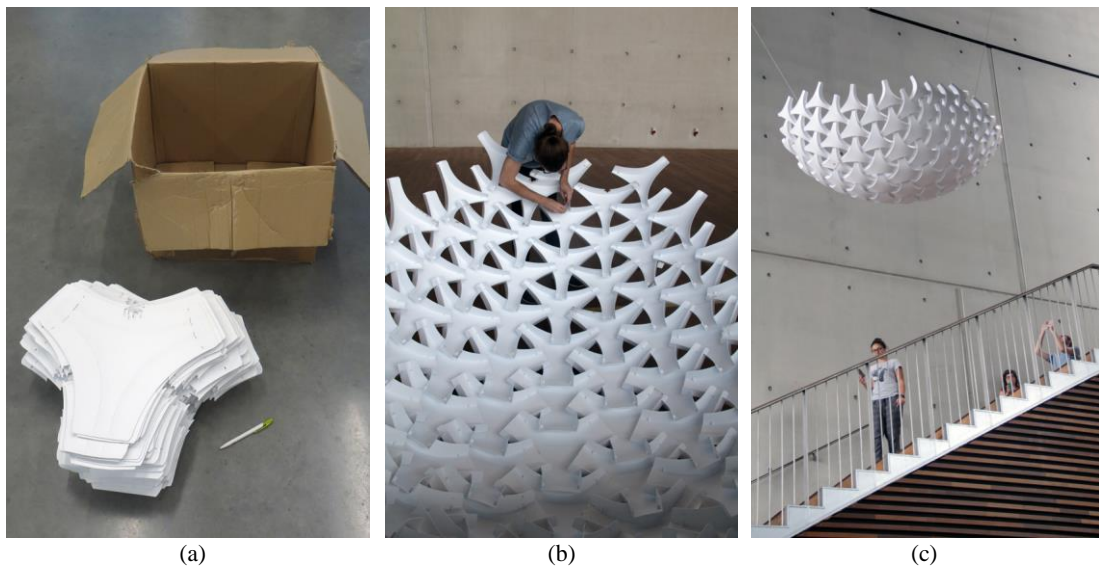

| (a) | (b) | (c) |

Figure 1: (a) Compact, flat-stacked configuration, (b) rapid assembly without screws or bolts, (c) Undulatus pavilion hanging at the IASS2015 expo.

2. Geometric and parametric modelling of a curved-line folding weave pattern

The primary concept behind the Undulatus structure is a kit-of-parts system composed of curved-line folding components. Each of these components has a triangle-like shape with three curved fold lines, separating a middle top part with three legs from three flanges (figure 2). Regular components (figure 2a) show a high degree of symmetry and work well for spherical geometries. For non-spherical surfaces, components with irregular and asymmetrical dimensions and fold lines are required (figure 2b), as this increases the number of available design parameters. Consequently, the designer has greater freedom to manipulate the geometry of each component individually and of the resulting overall structure.
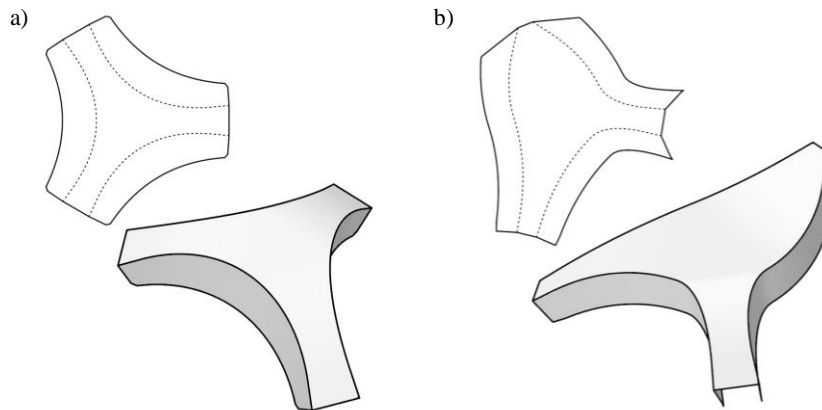

Figure 2: (a) A regular and (b) irregular basic curved-line folding component
in a flat and a bent state.

The large freedom in design and irregularity of components asks for digital design tools in order to explore the geometrical potential of this structural concept in an efficient, intuitive and interactive manner. We've developed such tools using the 3D modelling software Rhinoceros® and its parametric design plug-in Grasshopper® (Robert McNeel & Associates [7,8]), as well as the live-physics engine Kangaroo (Piker [9]).

The manner of modelling the components in these tools is based on a simulation strategy proposed by Tachi and Epps [10]: each component is discretised using the curved fold lines and the rulings of the curved surfaces. The resulting mesh consists of planar quadrangular faces and one triangular face in the centre of the component (figure 3). As such, the curved-line folding component is reduced to a rigid foldable plate component that forms a mechanism with a single degree of freedom. This simplification allows to instantly model the three-dimensional geometry of a component with a given set of fold lines for a given fold angle δ (representing the degree of bending) using a set of mathematical equations based on spherical trigonometry. The derivation of this set of equations for four-valent vertices is given in Huffmann [4] and its use demonstrated in Vergauwen *et al.* [11]. The equations for the five-valent vertices of the central triangular face are derived in a similar manner. Applying this mathematical description ensures that the sum of the angles at each interior vertex of the mesh equals 2π, thus ensuring that the three dimensional geometry is flat foldable.

The fold lines discretising the curved surfaces represent their rulings and thus determine how the component deforms. We've chosen them to be parallel, resulting in cylindrical curvature without torsion, which corresponds well with the behaviour of our test models. As a result, the fold lines of each leg of the component are symmetrical about the central axis of the leg (figure 3).
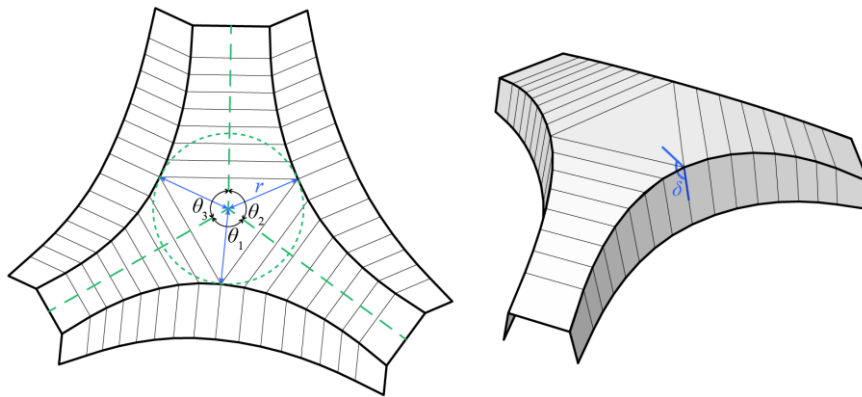

Figure 3: Mesh of a component with circumradius r of the central triangle and angles θ_i between the legs in the flat state and in a bent state (with fold angle $\delta = 105°$).
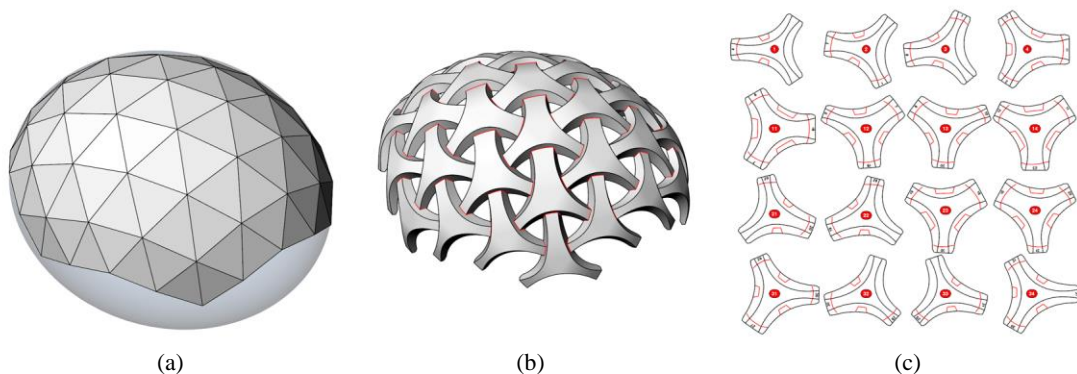

| (a) | (b) | (c) |

Figure 4: Three main design steps of the digital tool used to model the Undulatus pavilion:
(a) modelling the base grid; (b) populating the grid with curved-line folding components;
(c) output of the components in their flat state with intersection lines and labels.

In the parametric design tool used to model the Undulatus pavilion three main design steps can be distinguished as shown in figure 4. During the first step the geometry of the base grid is modelled using Kangaroo. This plug-in for Grasshopper® contains a live optimisation module based on a set of user-defined geometric constraints and target values, through which it is possible to sculpt the geometry and the base grid in an interactive and hands-on manner. Due to the desired weave pattern, this input grid must be triangulated and its interior vertices must all have a valency of six. This requirement sets constraints on the shape of the base surface, as it will for example not be possible to

envelope a full sphere with such a pattern in a continuous manner. Once a suitable grid is obtained, it is in a next step populated by the curved-line folding components by placing one component at each interior node of the grid (figure 4b). This step makes use of a custom-made optimisation routine that determines the values r and θ_i for each component (figure 3) so that it optimally fits in its location of the grid. The location of a component in the base grid, and thus the geometry of this grid, therefore determines its size and the lengths and directions of its legs. Additionally the user can freely define a range of other design parameters for each component individually, independently from the chosen input grid (marked in figure 5): the fold angle δ (giving the degree of bending), the shape of the curved fold lines (e.g., circular, parabolic or hyperbolic), the leg width of each leg (in order to control the size of the holes in the pattern), the depth of the weave effect for each leg, the width of the flanges and the extension of the legs beyond the intersection line (important for the design of the joints). As such, the user can extensively manipulate the structure to obtain the desired shape (figure 6).
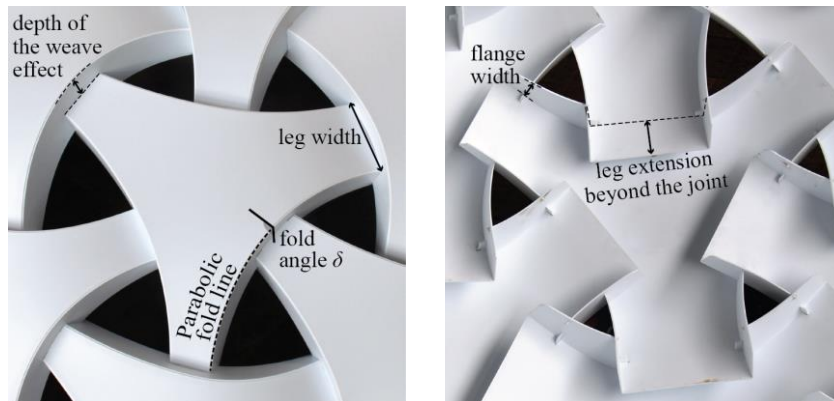

Figure 5: Design parameters that can be freely chosen by the designer for each leg of each component independently, marked on a composition of seven components viewed from above and below.
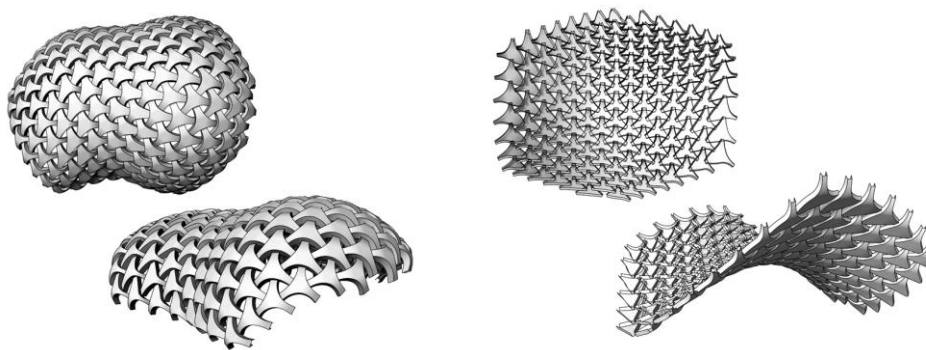

Figure 6: Two other possible outcomes of the digital design tool making use of the curved-line folding weave pattern, in top and perspective view.

In the final step, the script determines the intersection lines between the components and outputs their flat geometry, automatically labelled for easy and correct assembly (figure 4c). Once provided with the detailing (which depends on the material used and the design of the connections), this output can be sent to a CNC cutting machine for fabrication.

The final base grid of the Undulatus pavilion has an ellipsoidal shape. It consists of 99 components, covering a total area of about 8m². The leg widths vary between 3 and 14cm to obtain a dense centre that opens up towards the edges (figure 7).
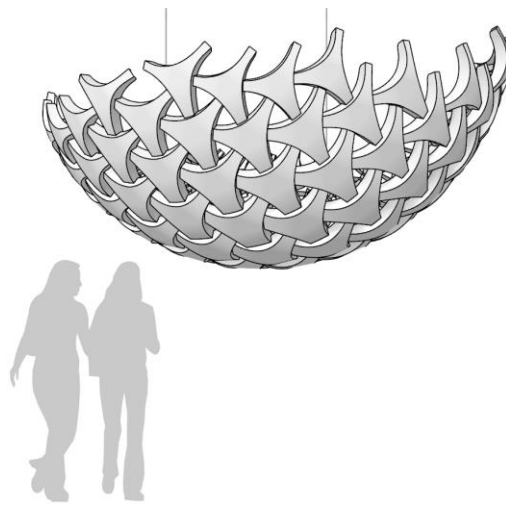

Figure 7: Digital simulation of the Undulatus pavilion using a parametric Grasshopper model.

3. Materialisation and detailing of the shell structure

As explained in the previous paragraph, the geometry of the pavilion is digitally modelled as if the components all form stress-free mechanisms that can easily retain their shape by simply locking the single degree of freedom. This simplification was necessary in order to obtain a fast explorative and interactive design tool with integrated optimisation routine. However, due to interlocking and stress-stiffening effects, the material behaviour in bending-active structures has a large impact on the final shape and load-bearing behaviour of the structure. In order to simulate a more precise structural behaviour, material properties and residual stress due to the elastic deformation of the components, should be taken into account in the digital model. Kangaroo allows inputting physical information in the geometric simulation in Grasshopper®. Yet, structural feedback is not included. On the other hand, modelling the elastic deformations in finite element software provides the necessary structural information but is very time consuming when used in the preliminary design phase. Therefore, a series of physical models and mock-ups was used to quickly gain insight in the material choice, material thickness and global stiffness of the shell structure (figure 8). Furthermore, the use of different materials entailed important variations in the realisation of the connections and fold lines. Where polypropylene and other plastics allowed elastic hinging along engraved crease lines, cardboard and wooden models required full cuts and reattachment, e.g., by sewing, textile hinges or tension straps. Another advantage of polypropylene (PP) is its high resistance to fatigue making it a very suitable

material for repetitive folding and bending. For the final structure white polypropylene sheets with a thickness of 1,8 mm are chosen. By means of the parametric digital model the average size of the components could easily be adapted to the retail size of PP sheets in order to get a maximum of components out of each sheet and have a minimum of material waste.
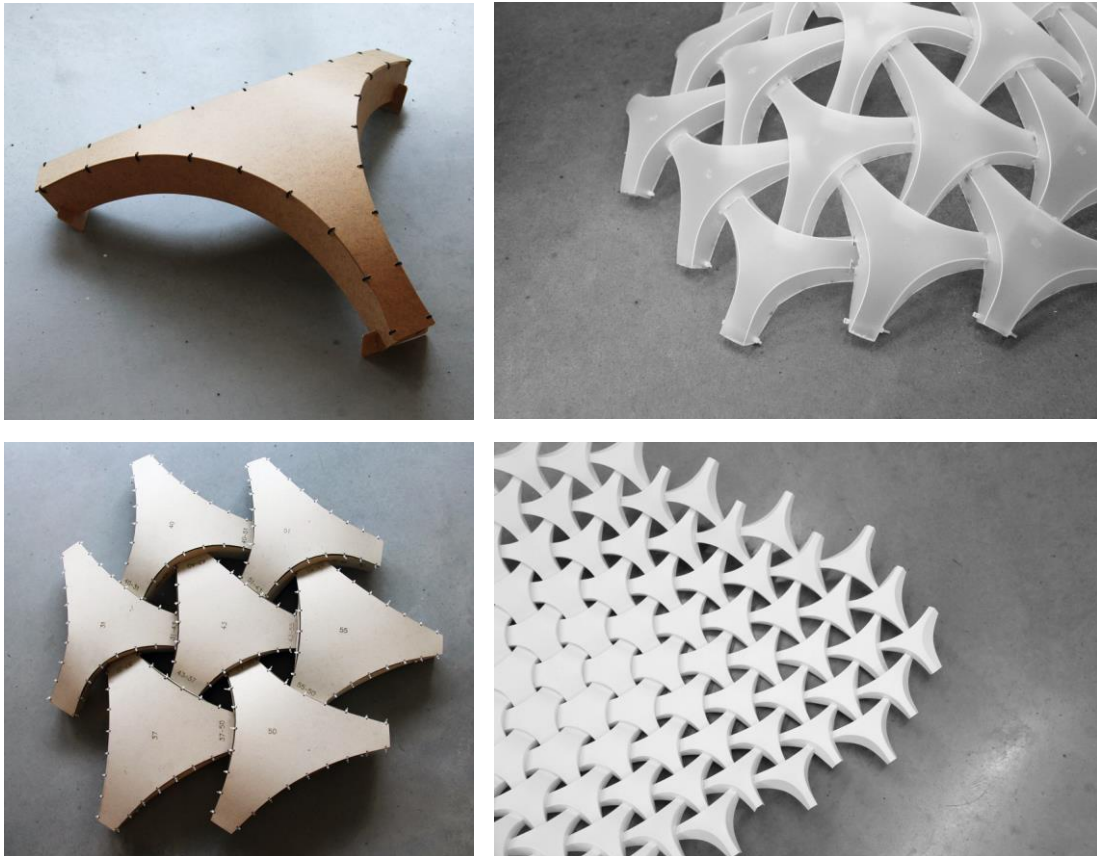

Figure 8: Physical models and mock-ups informed the geometric model of the Undulatus.

As with all kit-of-parts structures, and in extension all lightweight and transformable structures, the nodes are the most critical points. By realising the connections through small cutouts in the components, no additional connection pieces are needed, supporting the minimalistic nature of the system and the ease of assembly. By means of different physical models various designs of this interlocking connection where tested and after several iterations a pleasing solution was found. Figure 9a shows one of the components with integrated connections: one at the leg, another on the flange. By sliding the flange over the leg of a neighbouring component, the connection locks the folding angle of the leg and —after connecting all three legs— of the entire component. Two protrusions on the flange connection slide into the leg to lock translational and rotational movement (figure 9).

Figure 9: (a) Integrated connections restrain the components in a (b) slide-and-click system.

4. Digital fabrication and rapid assembly

The primary advantage of the curved-line folding principle behind the Undulatus pavilion is the ability to fabricate all the components from flat sheet material (figure 10). This increases the compactness of the flat-stacked elements, but also offers interesting solutions in digital fabrication techniques, such as CNC laser cutting. In fact, the entire pavilion was cut from thin, white polypropylene sheets. With a material thickness of only 1,8mm the total weight, comprising all 99 components, remains just under 30kg. Surface engravings serve as fold lines, which act as elastic hinges. Thanks to their high toughness, the polypropylene sheets have a high resistance to tearing, allowing local stresses and repeated folding and unfolding. All of the 99 components fit in one box of 45x50x40 cm (figure 1a), which ensures a very compact and easy transportation. Thanks to the simple, integrated connections, the kit of components could be assembled in about two and a half hours (figure 11). Since this concerned the first time assembly and included cleaning the components, this timespan can easily be reduced in the future.

In order to hang the Undulatus pavilion from the ceiling of the 'Muziekgebouw aan't IJ' in Amsterdam, plywood plates are connected to three of the components and supplied with an eye bolt and a U-shackle (figure 12). Three cables with a diameter of 4mm are used to ensure a stable suspended structure. Figure 13 shows the pavilion before suspension and in its final position.
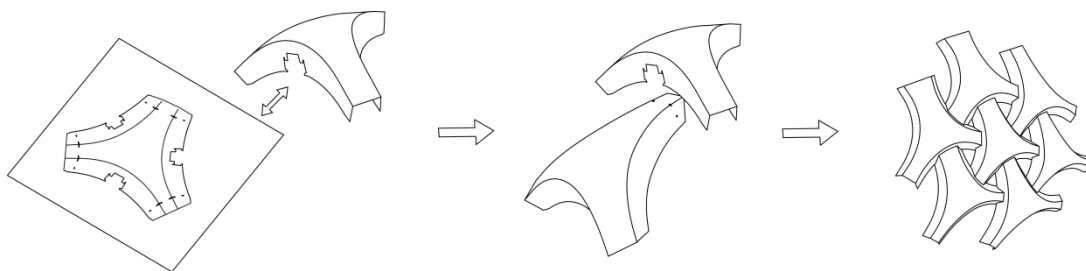

Figure 10: Three-step production and assembly: cutting the components, bending and locking them by coupling action and creating a curved, weave pattern.

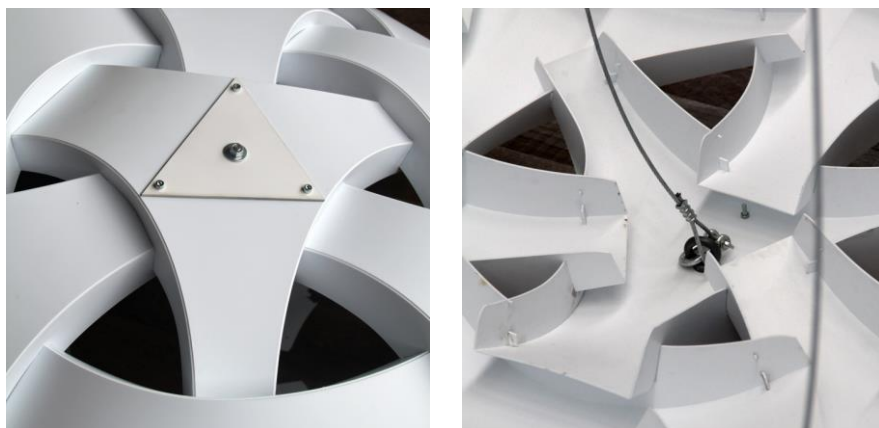Figure 11: Demonstration of the rapid assembly of the pavilion.

Figure 12: Detail of the plywood plate attached to the component to allow the cable connection.
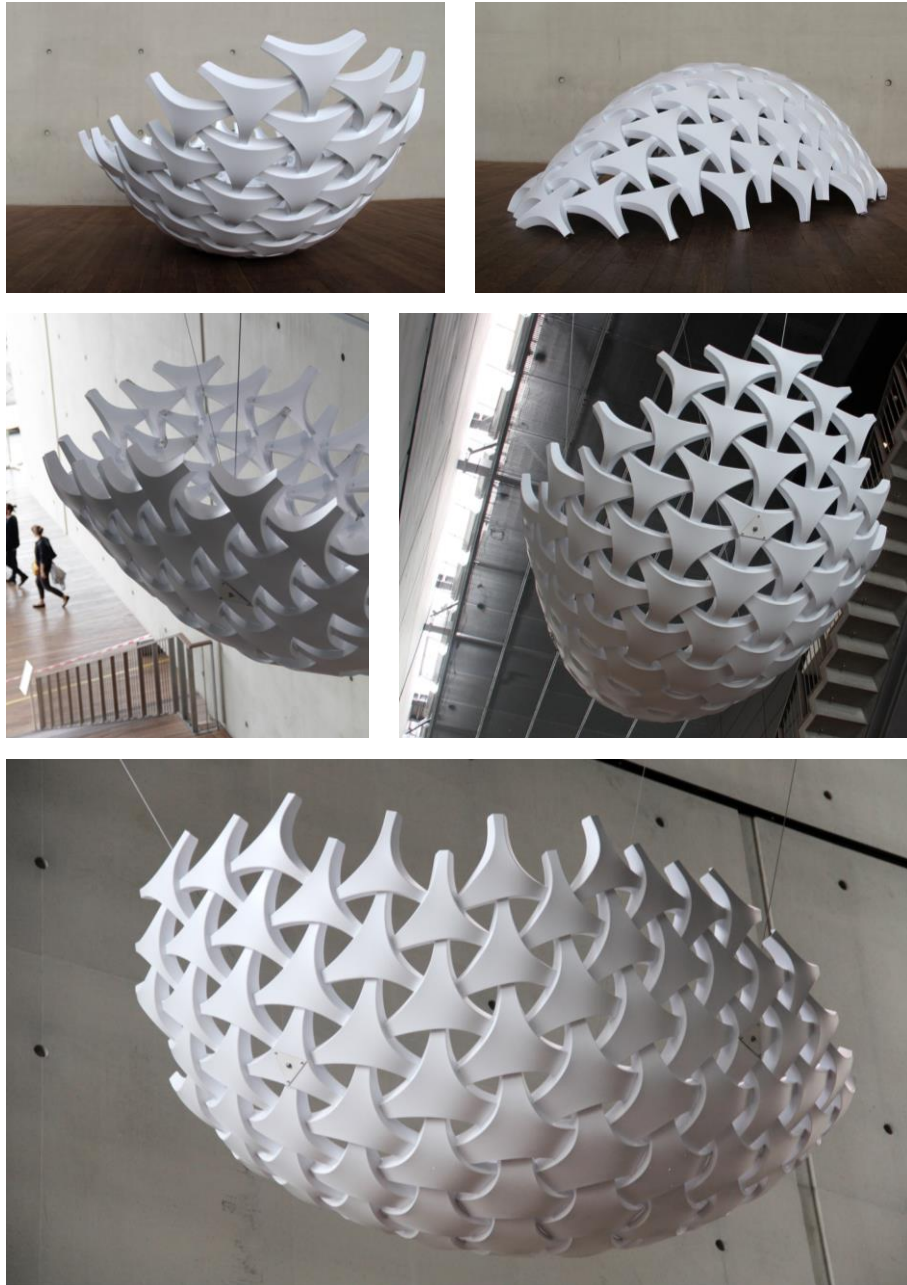
Figure 13: The Undulatus pavilion before suspension showing shell-like behaviour (top) and after suspension in the 'Muziekgebouw aan't IJ' in Amsterdam.

5. Conclusions

The Undulatus research pavilion demonstrates how curved-line folding can be used as a technique to design a very mobile and easily demountable bending-active shell structure. The pavilion consists in its compact state of 99 flat-folded components that all fit in a box of 45x50x40 cm. Due to the easy-to-handle, reversible connections the system has the advantage of high speed of assembly (about 2,5 hours), low weight (less than 30 kg), demountability and reuse of its components.

Although the digital model did not take into account material properties, it proved to be very valuable to explore a wide range of geometrical designs in an efficient and interactive manner. By observing small scale models the input parameters for the digital model were optimized to correspond better to reality. Since the shape of the final structure corresponds very well with the digital one, it can be concluded that this approach was a success. Of course, in order to gain more insight in the structural behaviour of the shell structure a FEM-analysis should be performed.

Typical structural applications of curved-line folding imply permanent plastic deformations at the hinge, which means that the artefact cannot return to its initial flat state. To our knowledge, the Undulatus research pavilion is the first structural application of curved-line folding in which the folding process is reversible and used to allow compact transportation and reuse of the components.

Acknowledgement

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